Controls on the channel width of rivers: Implications for modeling fluvial incision of bedrock

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ABSTRACT

On the basis of the Manning equation and basic mass conservation principles, we derive an expression for scaling the steady-state width (W) of river channels as a function of discharge (Q), channel slope (S), roughness (n), and width-to-depth ratio (α): $W = [\alpha(\alpha + 2)^{2/3}]^{3/8}Q^{3/8}S^{-3/16}n^{3/8}$. We propose that channel width-to-depth ratio, in addition to roughness, is a function of the material in which the channel is developed, and that where a river is confined to a given material, width-to-depth ratio and roughness can be assumed constant. Given these simplifications, the expression emulates traditional width-discharge relationships for rivers incising bedrock with uniformly concave fluvial long profiles. More significantly, this relationship describes river width trends in terrain with spatially nonuniform rock uplift rates, where conventional discharge-based width scaling laws are inadequate. We suggest that much of observed channel width variability in river channels confined by bedrock is a simple consequence of the tendency for water to flow faster in steeper reaches and therefore occupy smaller channel cross sections. We demonstrate that using conventional scaling relationships for channel width can result in underestimation of stream-power variability in channels incising bedrock and that our model improves estimates of spatial patterns of bedrock incision rates.

Keywords: fluvial geomorphology, tectonic geomorphology, channel width, river incision, landscape evolution.

INTRODUCTION

The role of bedrock-channel incision on the evolution of mountainous topography has become a central concept in geomorphology (e.g., Seidl and Dietrich, 1992; Burbank et al., 1996). Considerable attention to rivers incising bedrock in tectonically active landscapes (e.g., Whipple and Tucker, 1999) has led to the use of river morphology in interpreting the scale, magnitude, and timing of rock uplift, for which other evidence is often sparse or equivocal (e.g., Lavé and Avouac, 2001; Finlayson et al., 2002; Kirby et al., 2003). Collectively, this work has highlighted the stream-power model of river incision as a valuable tool for exploring the dynamics of fluvial erosion of bedrock.

The stream-power model generally casts bedrock incision rate, E, as a function of river slope, S, and river discharge, Q: $E = kQ^mS^n$, where k represents bedrock-specific erosivity, and m and n are empirically determined or selected on the basis of the hypothesized control on incision rate, e.g., power or shear stress (Whipple and Tucker, 1999). Estimation of river power per unit bed area or shear stress requires direct knowledge of channel width, typically approximated as bank-full channel width. However, spatially continuous width measurement necessitates high-resolution imagery and/or labor-intensive ground surveying. For this reason, models of bedrock incision often do not treat width explicitly, but instead rely on the assumption that channel width is a power-law function of discharge where $W \propto Q^b$. This substitution subsumes width variations into the exponent m on Q in the stream-power model. Substantial empirical work suggests that discharge-based width-scaling relationships are valid for alluvial rivers and that $b \approx 0.5$ (e.g., Knighton, 1998). Examples where these relationships have been evaluated for bedrock channels typically exhibit exponents on area or discharge of 0.3–0.5 (e.g., Whipple, 2004).

Field-based studies provide evidence for the alternative view that channel width varies locally, much like channel slope does, in association with spatial changes in rock uplift rate and erodibility. Lavé and Avouac (2001) and Montgomery and Gran (2001) demonstrate downstream narrowing of river channels in bedrock associated with a downstream increase in rock uplift rate and bedrock hardness, respectively. Additionally, Duvall et al. (2004) showed that variation in channel width, as well as slope, can account for inferred differences in long-term fluvial incision rates between two neighboring rivers undergoing different uplift. These studies show that simple scaling relationships between width and discharge alone are not adequate in precisely those situations where it is most interesting to be able to estimate bedrock incision rates; i.e., where rates of uplift or rock erodibility vary spatially. Additionally, in an effort to provide a theoretical basis for the empirical equations of downstream hydraulic geometry, Griffiths (2003) derived a series of analytical expressions that relate channel width, roughness, depth, slope, and discharge. However, because the analysis relies on the constraint that width scales linearly with downstream distance, the Griffiths model cannot address the downstream narrowing of channels. Generally, it remains unknown whether adoption of the traditional assumptions of hydraulic geometry has hindered understanding of the coupling between bedrock incision and tectonic processes.

We derive a simple relationship for the steady-state scaling of channel width. We then evaluate the proposed model for three rivers that are incising bedrock under both uniform and spatially variable rock uplift. Under the simplifying assumption of constant channel width-to-depth ratio and Manning's n, we compare how conventional width scaling laws and our proposed model predict spatial patterns in erosive potential when incorporated in a stream-power calculation. We conclude that bedrock channel width varies with both discharge and river slope, and that scaling channel width with only discharge, as is common, underestimates unit stream power in areas where rivers steepen downstream.

MODEL

The Manning (1891) equation, which is widely accepted as an empirical flow law for rough, steady, and uniform channel flow, states that the cross-section average water velocity, U, is given by:

$$U = R^{2/3}S^{1/2}/n, \tag{1}$$

where R is hydraulic radius, S is bed slope, and n (Manning's n) is an empirical roughness coefficient. The hydraulic radius of the flow is its cross-section area, A, divided by the wetted-perimeter, W_p. For a rectangular channel where bank-full width, W, and depth, D, are related by the width-to-depth ratio $\alpha = W/D$ (referred to hereafter as α), area and wetted perimeter are easily rewritten in terms of bank-full channel width

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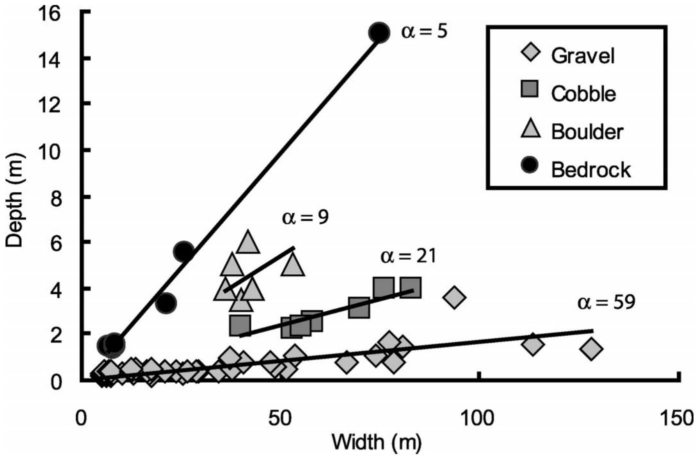

Figure 1. Plot of α for different dominant channel substrates. Gravel data are from Yellowstone River, Wyoming (Leopold and Mattock, 1953), and largest bedrock width data point is from LiWu River, Taiwan (Hartshorn et al., 2002). Other data are from field surveys in Cascades of Washington State. While gathered from various locations, all bedrock data are from channels incised in high-grade metamorphic or granitic rocks.

$$W_p = W + 2D = W(2/\alpha + 1), \tag{2}$$

and

$$A = W^2/\alpha. \tag{3}$$

Substituting equations 2 and 3 for R in equation 1 and simplifying yields

$$U = (\alpha + 2)^{-2/3} W^{2/3} S^{1/2}/n. \tag{4}$$

It is straightforward to show that equation 4, although derived here for a rectangular channel, is valid for all rivers where W and R are linearly related, i.e., for channels where α is constant. In natural channels, α appears to be relatively constant for a given channel-bed material (Fig. 1), suggesting that the assumption of constant width-to-depth ratio is an acceptable simplification for a channel developed in a particular material. From mass conservation, the flow discharge can be expressed as $Q = UA$. Multiplying equations 4 and 3 and solving for W therefore leads to

$$W = [\alpha(\alpha + 2)^{2/3}]^{3/8} Q^{3/8} S^{-3/16} n^{3/8}. \tag{5}$$

Equation 5 thus states that channel width naturally increases with discharge (and therefore drainage area), as is widely recognized. The equation explicitly incorporates the effects of α and n, the latter of which by impeding flow increases cross-section area. The equation also states that width can decrease with increasing channel slope, creating the potential for narrowing of channels where rivers steepen, e.g., where they enter regions with faster rock uplift rates or more resistant rock (e.g., Seeber and Gornitz, 1983; Kirby et al., 2003).

METHODS

We tested our model predictions for two relatively simple, concave-up streams incising bedrock and undergoing spatially uniform uplift. Oat Creek and Kinsey Creek drain the coastal King Range in northern California and have each been intensively surveyed for channel width (Snyder et al., 2003). We also tested our model against data from the bedrock-channel part of the more complex Yarlung Tsangpo River in southeastern Tibet where it traverses a major antiformal uplift in the eastern Himalayan syntaxis. As it crosses this structure, the river steepens and narrows considerably within its bedrock gorge.

We restricted our analysis of the Yarlung Tsangpo to the region where it is clearly incising bedrock and sampled width every 100 m from a continuous map of channel width created by digitizing channels from 28.5 m pixel resolution Landsat 7 Thematic Mapper images of the river at high flow. Channel elevation profiles and slopes were derived from the 3-arc second (~90 m) Defense Mapping Agency Digital Terrain Elevation Data (DTED) for Asia. Mean annual discharge was obtained by routing annual accumulated Tropical Rainfall Measuring Mission (TRMM; http://trmm.gsfc.nasa.gov/) satellite-derived rainfall on the 30-arc second GTOPO 30 digital elevation model (DEM) of Asia. Estimates of discharge using TRMM data have been shown to agree with gauged mean annual discharges for 21 Himalayan catchments to within 15% (Anders et al., 2005). Due to the relatively coarse DEM used for the Yarlung Tsangpo, it was necessary to segment the river into 10 km reaches in order to remove noise in the river elevation profile. For each reach, mean elevation, mean channel width, and slope were calculated.

We used Snyder et al.'s (2003) field-measured bank-full width data for Oat Creek and Kinsey Creek. Elevation and slope data were derived from 10-m-resolution U.S. Geological Survey digital elevation data as reported in Snyder et al. (2003). We calculated mean width, elevation, and slope for every 3 data points in the data set, resulting in mean values spaced at roughly 100 m intervals. Mean annual discharge was estimated for Oat Creek and Kinsey Creek by applying the relationship between mean annual discharge and drainage area for nearby Honeydew Creek (Snyder et al., 2003).

Errors in slope data for all rivers were calculated using the root mean square error of 7 m for the 10-m-resolution DEM and 18 m for 3-arc-second-resolution DEM (EROS Data Center, 2000; Gesch and Larson, 1998). These uncertainties were propagated through the various calculations of mean width and mean stream power below and are represented in the figures. It is important to keep in mind that the uncertainties shown in the figures apply only to values averaged over 10 km for the Yarlung Tsangpo, and roughly 100 m for the rivers in the King Range.

MODEL EVALUATION

Like alluvial rivers, many rivers incising bedrock also exhibit relatively simple, concave-up elevation profiles, where slopes decrease steadily in a downstream direction. For this class of rivers, elevation profile data can be fit into the form

$$S = kQ^{-\theta}, \tag{6}$$

where θ is referred to as the concavity index (Flint, 1974). Substitution of equation 6 into 5 yields the following expression for the dependence of the width of a river on concavity and discharge, with constant α:

$$W \propto Q^{3/8 + 3\theta/16} n^{3/8}. \tag{7}$$

For a typical concavity index of ½ and constant roughness, equation 7 yields

$$W \propto Q^{15/32} \approx Q^{0.47}. \tag{8}$$

Hence for rivers with uniform concavities, the model accounts for the discharged-based downstream width relations typically observed for alluvial channels. More significantly, the similarity of our relationship to traditional width empiricisms is confirmed for bedrock channels after plotting measured channel width, channel width determined from equation 5 with constant n and α ($kQ^{3/8}S^{-3/16}$), and channel width determined from assuming width scales with $Q^{1/2}$ along Kinsey Creek and Oat Creek in northern California (Fig. 2). Notably, the simplified version of equation 5 is indistinguishable from a simple relationship in which width scales only with $Q^{1/2}$.

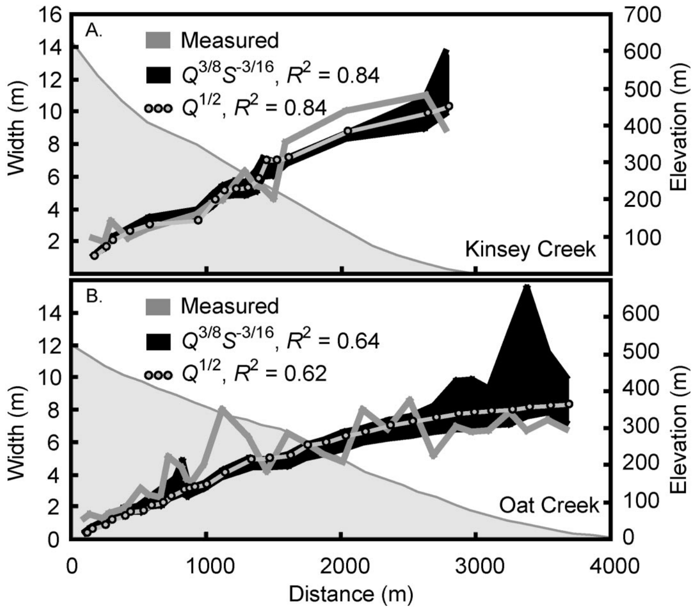

Figure 2. River long profiles and comparisons of measured channel width (Snyder et al., 2003), channel width derived from a $Q^{1/2}$ relationship, and channel width derived from equation 5 for Kinsey Creek (A) and Oat Creek (B), California, USA. Uncertainties due to errors in width measurement and digital elevation model–derived river slopes are represented by line thickness.

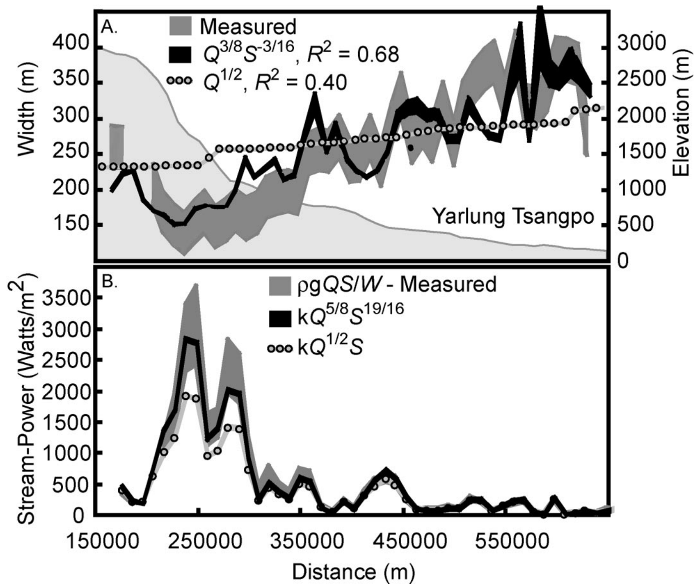

Figure 3. A: River long profile and comparison of measured channel width, channel width derived from a $Q^{1/2}$ relationship, and channel width derived from equation 5 for Yarlung Tsangpo, southeast Tibet. B: Comparison of unit stream power ($\rho gQS/W$) for Yarlung Tsangpo using measured channel width, width derived from equation 5, and width derived from $Q^{1/2}$ relationship. Uncertainties due to errors in width measurements and digital elevation model–derived slopes are represented by line thickness for both A and B.

Equation 5 also predicts that anomalously steep reaches will have higher water velocities—provided that changes in channel bed roughness do not offset those in slope—and, in order to conserve water flux, smaller cross sections. Under the assumption of constant α, channel width and depth are proportional to the square root of cross-section area, and so these reaches should be both shallow and narrow. Rivers incising bedrock in tectonically active or lithologically variable regions frequently lack consistent and uniform concavities, and local convexities are commonplace (Seeber and Gornitz, 1983; Kirby et al., 2003). In such locales, equation 5 indicates that simple width-discharge scaling relations should lose their predictive power.

To evaluate this analytical prediction in more tectonically complex terrain, we applied our model to the Yarlung Tsangpo River. For the Yarlung Tsangpo, we calculated linear regressions (forced through the origin) of measured channel width vs. $Q^{1/2}$ and of measured channel width vs. $Q^{3/8}S^{-3/16}$ (equation 5 with constant n and α). For the Yarlung Tsangpo, regression of width against $Q^{3/8}S^{-3/16}$ yielded better fits ($R^2 = 0.68$) than did a model that scaled width with only $Q^{1/2}$ ($R^2 = 0.40$) (Fig. 3A). The extent to which our model matches the data supports the idea that channel shape tends toward self-similar adjustments when confined to bedrock. These changes depend primarily on discharge and slope, which are externally modulated by factors such as regional base level, rock uplift rate, bedrock resistance to erosion, and climate.

MODELING RIVER INCISION

River long profile analyses, which typically rely on width-discharge scaling relationships, have become an important tool for inferring spatial and temporal patterns in rates of bedrock incision and, where steady state is assumed, rates of rock uplift. In order to explore the sensitivity of river incision models to different methods for estimating channel width, we compared how conventional width scaling laws and our model predict spatial patterns in unit stream power (Fig. 3B). For the Yarlung Tsangpo, unit stream power ($\Omega = \rho gQS/W$) was calculated using mean annual discharge, slope, and channel width obtained in three ways: (1) measured; (2) determined from a conventional width relationship of the form $W \propto Q^{1/2}$, which yields an expression of the form $kQ^{1/2}S$; and (3) determined from equation 5, which with constant n and α, yields an equation that takes the form

$$\Omega = kQ^{5/8}S^{19/16}. \tag{9}$$

For the Yarlung Tsangpo, use of a simple width-discharge scaling law results in as much as a 40% underestimate of unit stream power along the sections of the river that steepen downstream. In general, such an approach also tends to damp the spatial variability in erosive power because it does not reflect the tendency for channels to narrow where they steepen. Equation 5 comes closer to describing the full downstream variability in the width of the Yarlung Tsangpo and hence the actual unit stream power (Fig. 3B). Our model thus has important implications for fluvial responses to variable rock uplift and lithology, which are of particular interest in most active orogens. Because width appears to narrow as slope increases, a channel requires less of a change in slope (and therefore fluvial relief) than it would if slope, alone, responded to a particular forcing such as rock uplift rate, bedrock resistance to erosion, or climate. This tendency for channels to narrow as they steepen thus provides a negative feedback on fluvial relief change and results in the nonlinear exponent on slope in equation 9.

DISCUSSION

Equation 5 highlights the relative complexity of modeling channel width when flow velocity is considered. However, all stream-power incision models make implicit assumptions about velocity. For example, Whipple and Tucker (1999) assumed that roughness is constant by inserting the dimensionless friction factor, C_f, from their momentum equation into the rate constant, K, in their erosion equation. In addition, in their derivation width is assumed to vary only with discharge. These constraints force flow depth, and therefore α, to accommodate all variation in cross-sectional area due to changes in velocity dictated by the momentum equation. Although this model was clearly not developed with the intention of predicting reach-scale morphology, it provides an example of how stream-power models make implicit requirements of the morphology of rivers. In order to explore the particular phenomenon of width scaling observed in bedrock channels, we have explicitly coupled width and depth in our analysis, and thereby allow changes in cross-sectional area to be accommodated by both depth and width. While we provide justification that α remains relatively constant for channels confined to a particular material (Fig. 1), we acknowledge the fundamental control that our choice of treating the width-to-depth ratio imposes on the scaling of behavior of channel width in our analysis. Given the lack of a clear consensus on the controls on width-to-depth ratio in natural channels, we stress that other ways of scaling α can easily be accommodated in our framework. Nonetheless, equation 5 should be applied cautiously, as it requires that R (hydraulic radius) and W (channel width) be linearly related.

From Figure 1 it is also clear that significant changes in channel width are to be expected at boundaries between different types of channels, for example where a river transitions from alluvium to bedrock. Such width transitions do not result from flow velocity changes alone, and it is for this reason that we avoid alluvial reaches in our analysis. We speculate that the systematic changes in α shown in Figure 1 reflect different critical shear stresses for mobilization or erosion of channel boundaries developed in different materials. Specifically, bedrock channels can support much higher wall shear stresses than gravel channels. Hence a river can likely maintain a narrower channel in bedrock than in gravel at the same discharge.

Because channel roughness, bed material, and caliber are inextricably linked, it is difficult to consider any of these factors independently, particularly for channels with mobile beds. Without knowledge of Manning's n, we have assumed in all of our regressions and modeling that n is constant. As noted earlier, all stream-power formulations make the same implicit assumption of constant roughness. However, there is significant work that suggests that Manning's n may scale strongly with bed slope in alluvial rivers. For example, Dingman and Sharma (1997) reported that n varies with bed slope to a power of 0.3–0.4 in alluvial channels. Adoption of such a relation for roughness would effectively cancel out any slope dependency in our model, thus reducing width to a simple function of discharge and α. This further simplification provides a good explanation for the observation that significant narrowing and changes in flow velocity are generally absent in alluvial channels (Leopold and Mattock, 1953), but it is unable to explain the downstream narrowing observed in natural channels confined in bedrock.

Whereas alluvial channel roughness can vary as bed coarsening occurs at higher shear stresses, a channel bounded by bedrock lacks an obvious mechanism to change boundary roughness as boundary shear varies. Therefore, we suggest that for the end-member case of a channel truly confined within bedrock boundaries, increases in flow velocity and narrowing will tend to occur where channels steepen because the flow feedbacks that operate in alluvial channels are suppressed. On the Yarlung Tsangpo, it appears that any downstream increases in roughness along the steeper reaches of the river are not sufficient to prevent flow acceleration and the $\sim$50% channel narrowing observed along the river, unless width-to-depth ratio systematically decreases downstream.

CONCLUSIONS

We have developed a scaling relationship for the width of river channels that depends on channel slope, river discharge, roughness, and channel width-to-depth ratio. A simplified version of this relation applied to longitudinally simple, uniformly concave bedrock rivers mimics traditional width-discharge relations that scale river width with only the square root of discharge. However, equation 5 is considerably more versatile, as it also describes river width trends in more complex terrain with spatial variations in rates of rock uplift.

Application of conventional discharge-based width scaling relationships to bedrock channels tends to underestimate erosive power along reaches that steepen downstream. On the Yarlung Tsangpo, unit stream power calculated with a common discharge-based power law for channel width is as much as 40% lower than estimates made from satellite-based width measurements and from our model. Our analysis indicates that modeling of bedrock channel incision would be improved simply by accounting for adjustments in channel width due to the tendency for water flowing faster through steeper reaches to occupy smaller channels.

ACKNOWLEDGMENTS

We thank Noah Snyder and Kelin Whipple for providing data for Oat Creek and Kinsey Creek, Kelin Whipple for providing elevation data for the Yarlung Tsangpo, Felix Ng for insightful discussion, and Douglas Burbank and Eric Kirby for perceptive reviews that improved the manuscript. Funding was provided by the Continental Dynamics Program of the National Science Foundation (grant EAR-0003561) and a National Science Foundation Graduate Research Fellowship to Finnegan.

REFERENCES CITED

Anders, A.M., Roe, G.H., Hallet, B., Montgomery, D.R., Finnegan, N.J., and Putkonen, J., 2005, Spatial patterns of precipitation and topography in the Himalaya: GSA Penrose Conference Special Paper on Climate, Tectonics, and Landscape Evolution: Boulder, Colorado, Geological Society of America (in press).

Burbank, D.W., Leland, J., Fielding, E., Anderson, R.S., Brozovic, N., Reid, M.R., and Duncan, C., 1996, Bedrock incision, rock uplift and threshold hillslopes in the northwestern Himalayas: Nature, v. 379, p. 505–510.

Dingman, S.L., and Sharma, K.P., 1997, Statistical development and validation of discharge equations for natural channels: Journal of Hydrology, v. 199, p. 13–35.

Duvall, A., Kirby, E., and Burbank, D.W., 2004, Tectonic and lithologic controls on channel profiles and processes in coastal California: Journal of Geophysical Research, v. 109, no. F3, doi: 10.1029/2003JF000086.

EROS Data Center, 2000, U.S. GeoData digital elevation models: U.S. Geological Survey fact sheet 040-00: http://erg.usgs.gov/isb/pubs/factsheets/fs04000.html (April 2000).

Finlayson, D.P., Montgomery, D.R., and Hallet, B., 2002, Spatial coincidence of rapid inferred erosion with young metamorphic massifs in the Himalayas: Geology, v. 30, p. 219–222.

Flint, J.J., 1974, Stream gradient as a function of order, magnitude, and discharge: Water Resources Research, v. 10, p. 969–973.

Gesch, D.B., and Larson, K.S., 1998, Techniques for development of global 1-kilometer digital elevation models, *in* Proceedings, Pecora Thirteen, Human Interactions with the Environment—Perspectives from Space, 13th, Sioux Falls, South Dakota, August 20–22, 1996: Bethesda, Maryland, American Society of Photogrammetry and Remote Sensing, CD-ROM, 1 disc.

Griffiths, G.A., 2003, Downstream hydraulic geometry and hydraulic similitude: Water Resources Research, v. 39, no. 4, 1094, doi: 10.1029/2002WR001485.

Hartshorn, K., Hovius, N., Dade, W.B., and Slingerland, R.L., 2002, Climate-driven bedrock incision in an active mountain belt: Science, v. 397, p. 2036–2038.

Kirby, E., Whipple, K.X., Tang, W., and Chen, Z., 2003, Distribution of active rock uplift along the eastern margin of the Tibetan Plateau: Inferences from bedrock channel longitudinal profiles: Journal of Geophysical Research, v. 108, no. B4, 2217, doi: 10.1029/2001JB000861.

Knighton, D.A., 1998, Fluvial forms and processes: London, Edward Arnold, 377 p.

Lavé, J., and Avouac, J.P., 2001, Fluvial incision and tectonic uplift across the Himalayas of central Nepal: Journal of Geophysical Research, v. 106, p. 26,561–26,591.

Leopold, L.B., and Maddock, T., Jr., 1953, The hydraulic geometry of stream channels and physiographic implications: U.S. Geological Survey Professional Paper 252, 57 p.

Manning, R., 1891, On the flow of water in open channels and pipes: Institute of Civil Engineers of Ireland Transactions, v. 20, p. 161–207.

Montgomery, D.R., and Gran, K.B., 2001, Downstream variations in the width of bedrock channels: Water Resources Research, v. 37, p. 1841–1846.

Seeber, L., and Gornitz, V., 1983, River profiles along the Himalayan Arc as indicators of active tectonics: Tectonophysics, v. 92, p. 335–367.

Seidl, M.A., and Dietrich, W.E., 1992, The problem of channel erosion into bedrock: Catena Supplement, v. 23, p. 101–124.

Snyder, N.P., Whipple, K.X., Tucker, G.E., and Merritts, D.J., 2003, Channel response to tectonic forcing; field analysis of stream morphology and hydrology in the Mendocino triple junction region, northern California: Geomorphology, v. 53, p. 97–127.

Whipple, K.X., 2004, Bedrock rivers and the geomorphology of active orogens: Annual Reviews of Earth and Planetary Science, v. 32, p. 151–185.

Whipple, K.X., and Tucker, G.E., 1999, Dynamics of the stream-power river incision model; implications for height limits of mountain ranges, landscape response timescales, and research needs: Journal of Geophysical Research, v. 104, p. 17,661–17,674.

Manuscript received 10 September 2004
Revised manuscript received 19 November 2004
Manuscript accepted 26 November 2004

Printed in USA