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# A systematic analysis of eight decades of incipient motion studies, with special reference to gravel-bedded rivers

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**Abstract.** Data compiled from eight decades of incipient motion studies were used to calculate dimensionless critical shear stress values of the median grain size, $\tau^*_{c_{50}}$. Calculated $\tau^*_{c_{50}}$ values were stratified by initial motion definition, median grain size type (surface, subsurface, or laboratory mixture), relative roughness, and flow regime. A traditional Shields plot constructed from data that represent initial motion of the bed surface material reveals systematic methodological biases of incipient motion definition; $\tau^*_{c_{50}}$ values determined from reference bed load transport rates and from visual observation of grain motion define subparallel Shields curves, with the latter generally underlying the former; values derived from competence functions define a separate but poorly developed field, while theoretical values predict a wide range of generally higher stresses that likely represent instantaneous, rather than time-averaged, critical shear stresses. The available data indicate that for high critical boundary Reynolds numbers and low relative roughnesses typical of gravel-bedded rivers, reference-based and visually based studies have $\tau^*_{c_{50}}$ ranges of 0.052–0.086 and 0.030–0.073, respectively. The apparent lack of a universal $\tau^*_{c_{50}}$ for gravel-bedded rivers warrants great care in choosing defendable $\tau^*_{c_{50}}$ values for particular applications.

## Introduction

Incipient motion of streambeds is a fundamental process with applications to a wide variety of research problems, such as paleohydraulic reconstructions [*Church*, 1978], placer formation [*Komar and Wang*, 1984; *Li and Komar*, 1992], canal design [*Lane*, 1955], flushing flows [*Milhous*, 1990; *Kondolf and Wilcock*, 1992], and assessment of aquatic habitat [*Buffington*, 1995; *Montgomery et al.*, 1996]. Regardless of whether one advocates equal mobility [*Parker et al.*, 1982], selective transport [e.g., *Komar*, 1987a, b], or some other style of sediment movement, most investigators use a standard or modified form of the critical Shields parameter to define incipient motion of a grain size of interest. The Shields parameter, or dimensionless critical shear stress, is defined as $\tau^*_{c_i} = \tau_{c_i}/(\rho_s - \rho) g D_i$, where $\tau_{c_i}$ is the critical shear stress at incipient motion for a grain size of interest, $D_i$; $g$ is the gravitational acceleration; and $\rho_s$ and $\rho$ are the sediment and fluid densities, respectively. Of particular interest for fluvial geomorphologists is determination of dimensionless critical shear stress values of the median grain size, $\tau^*_{c_{50}}$, for high boundary Reynolds numbers characteristic of gravel-bedded streams.

*Shields* [1936] demonstrated that $\tau^*_{c_{50}}$ of near-uniform grains varies with critical boundary Reynolds number, $Re^*_c$, and hypothesized on the basis of an analogy with *Nikuradse*'s [1933] findings that $\tau^*_{c_{50}}$ attains a constant value of about 0.06 above $Re^*_c = 489$ (Figure 1). The critical boundary Reynolds number is defined as $Re^*_c = u^*_c k_s/\nu$, where $u^*_c$ is the critical shear velocity for incipient motion ($u^*_c \equiv (\tau_c/\rho)^{1/2}$), $k_s$ is the boundary roughness length scale, and $\nu$ is the kinematic viscosity; *Shields* [1936] set $k_s = D_{50}$, the median grain size of the sediment. Although *Shields*' [1936] boundary Reynolds numbers differ from *Nikuradse*'s [1933], the general form of *Shields*' [1936] curve (Figure 1) is quite similar to *Nikuradse*'s [1933] curve, indicating regions of hydraulically smooth, transitional, and rough turbulent flow. The commonly quoted value of $\tau^*_{c_{50}} \approx 0.06$ for rough turbulent flow reflects a single data point within the overall swath of *Shields*' [1936] data (Figure 1).

There have been numerous additions, revisions, and modifications of the Shields curve since its original publication. *Shields* [1936], *Grass* [1970], *Gessler* [1971], and *Paintal* [1971] recognized that incipient motion of a particular grain size is inherently a statistical problem, depending on probability functions of both turbulent shear stress at the bed and intergranular geometry (i.e., friction angles) of the bed material, the latter being controlled by grain shape, sorting, and packing [*Miller and Byrne*, 1966; *Li and Komar*, 1986; *Kirchner et al.*, 1990; *Buffington et al.*, 1992]. Consequently, there is a frequency distribution of dimensionless critical shear stresses for any grain size of interest. Reanalyzing *Shields*' [1936] data and correcting for sidewall effects and form drag, *Gessler* [1971] reported $\tau^*_{c_{50}} \approx 0.046$ for a 50% probability of movement in rough turbulent flow. Without consideration of the probability of movement, *Miller et al*. [1977] arrived at a similar value of $\tau^*_{c_{50}} \approx 0.045$ for rough turbulent flow using compiled flume data from various sources. *Miller et al*. [1977, p. 507] employed data from "flumes with parallel sidewalls where flows were uniform and steady over flattened beds of unigranular, rounded sediments"; sidewall corrections were applied and each source used a consistent definition of incipient motion. Although *Miller et al*. [1977] used carefully selected data to ensure compatibility within their compilation, scrutiny of their data shows use of both uniform and nonuniform sediment mixtures, differing incipient motion definitions between studies, and in some cases bed load transport rates influenced by bed forms.

Using a larger data set and ignoring differences in sediment



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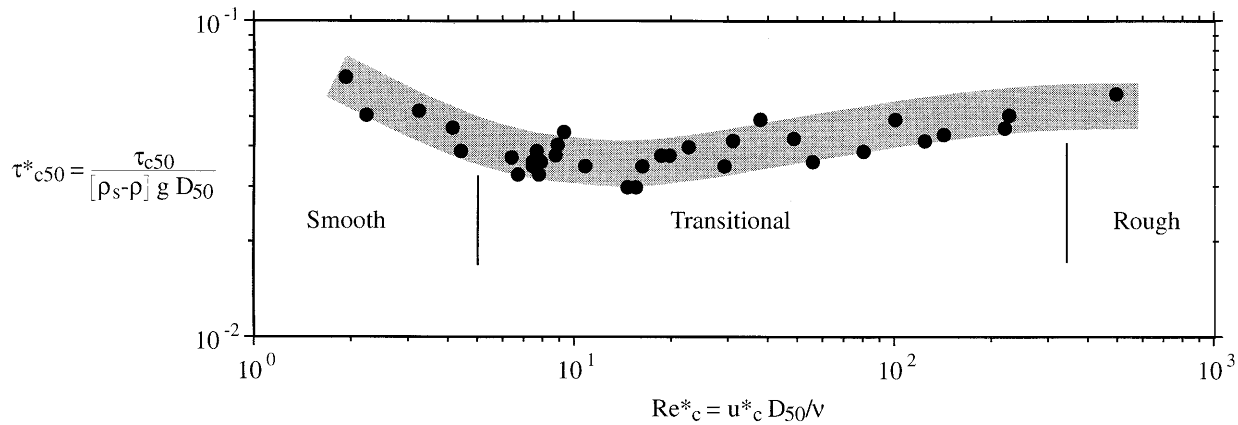


**Figure 1.** *Shields*' [1936] curve redrafted from *Rouse* [1939].

characteristics, channel roughness, or definition of incipient motion, *Yalin and Karahan* [1979, Figure 5] also report $\tau^*_{c_{50}} \approx 0.045$ for rough turbulent flow. They further demonstrate the existence of a second Shields curve for fully laminar flow, which for the same $Re^*_c$ values behaves differently than the traditional Shields curve derived from turbulent flow with variable hydrodynamic boundary conditions (i.e., smooth, transitional, or rough). The $\tau^*_{c_{50}}$ values reported by *Miller et al.* [1977] and *Yalin and Karahan* [1979] are average values of data sets with considerable scatter; in both studies individual $\tau^*_{c_{50}}$ values for rough turbulent flow range from about 0.02 to 0.065.

Previous compilations of $\tau^*_{c_{50}}$ values combine data derived from quite different experimental conditions and methodologies with little assessment of compatibility. Continued proliferation of incipient motion studies using new definitions of initial motion further complicates comparison and understanding of published studies. Although differences in experimental condition and methodology have been recognized and discussed [e.g., *Tison*, 1953; *Miller et al.*, 1977; *Carson and Griffiths*, 1985; *Lavelle and Mofjeld*, 1987; *Wilcock*, 1988, 1992b], their influence on reported $\tau^*_{c_{50}}$ values has not been well examined. Here we compile eight decades of incipient motion data and stratify calculated $\tau^*_{c_{50}}$ values by (1) initial motion definition, (2) choice of surface, subsurface or laboratory mixture median grain size, (3) relative roughness, and (4) flow regime, providing a systematic reanalysis of the incipient motion literature. We also evaluate the compatibility of different investigative methodologies and interpret the range of reported $\tau^*_{c_{50}}$ values.

## Data Compilation and Stratification

All available incipient motion data are summarized in Tables 1a–1e. Values of $\tau^*_{c_{50}}$, critical boundary Reynolds number ($Re^*_c$), and median grain size ($D_{50}$) are reported for each source, as well as experimental conditions and dimensionless critical shear stress equations where these are different than *Shields*' [1936]. Where available, the graphic sorting coefficient ($\sigma_g$ [*Folk*, 1974]), sediment density ($\rho_s$), and relative roughness ($D_{50}/h_c$, where $h_c$ is the critical flow height at incipient motion) are also reported. In many cases values of $Re^*_c$ and $\tau^*_{c_{50}}$ (or particular types of $\tau^*_{c_{50}}$, as discussed later) were not reported but could be calculated from the data and equations presented by the author(s); to be consistent with *Shields* [1936], we used $k_s = D_{50}$ when calculating $Re^*_c$. The graphic sorting coefficient is defined as $(\phi_{84} - \phi_{16})/2$, where $\phi_{84}$ and $\phi_{16}$ are the 84th and 16th percentiles of the grain size distribution expressed in units of the phi ($\log_2$) scale. Values of $h_c$ used to determine relative roughness were back-calculated from depth-slope products where sufficient data were reported. Detailed notes regarding both our calculations and the investigative procedures used by each source are presented by *Buffington* [1995] and are abbreviated in the appendix.

The data compiled in Tables 1a–1e are stratified by incipient motion definition. The four most common methods of defining incipient motion are: (1) extrapolation of bed load transport rates to either a zero or low reference value (Table 1a) [e.g., *Shields*, 1936; *Day*, 1980; *Parker and Klingeman*, 1982]; (2) visual observation (Table 1b) [e.g., *Gilbert*, 1914; *Kramer*, 1935; *Yalin and Karahan*, 1979]; (3) development of competence functions that relate shear stress to the largest mobile grain size, from which one can establish the critical shear stress for a given size of interest (Table 1c) [e.g., *Andrews*, 1983; *Carling*, 1983; *Komar*, 1987a]; and (4) theoretical calculation (Table 1d) [e.g., *White*, 1940; *Wiberg and Smith*, 1987; *Jiang and Haff*, 1993].

Dimensionless critical shear stresses determined from the first method are based on critical shear stresses associated with either a zero or low reference transport rate extrapolated from paired shear stress and bed load transport measurements. Values determined from this approach are sensitive to the extrapolation method [cf. *Parker and Klingeman*, 1982; *Diplas*, 1987; *Ashworth and Ferguson*, 1989; *Ashworth et al.*, 1992] and the particular reference transport value that is chosen [*Wilcock*, 1988].

Visual observation, used in the second method, is direct but can be subjective depending on one's definition of how much movement constitutes initial motion [e.g., *Gilbert*, 1914; *Kramer*, 1935; *Neill and Yalin*, 1969; *Wilcock*, 1988]. *Paintal* [1971] argues that there will always be some probability of grain movement as long as there is any fluid motion; hence the threshold of movement becomes a definitional construct [see also *Lavelle and Mofjeld*, 1987]. Standardized definitions of incipient motion have been proposed on the basis of the number of grains in motion, the area of bed observed, and the duration of observation [*Neill and Yalin*, 1969; *Wilcock*, 1988]; however, these definitions have not been widely adopted.

Competence functions, used in the third method, are sensitive to the size and efficiency of the sediment trap, sample size, sampling strategy, availability of coarse grain sizes, and curve-fitting technique [*Wilcock*, 1992b; *Wathen et al*., 1995]. Furthermore, the competence method is inappropriate for sediment that exhibits equal mobility, as the competence approach relies on selective transport [*Wilcock*, 1988, 1992b].

The fourth method utilizes simple force balance arguments to predict initial motion thresholds and is sensitive to model parameters such as grain protrusion, packing, and friction angle. In our analysis, $\tau^*_{c_{50}}$ values corresponding to these four methods of measuring incipient motion are symbolized as $\tau^*_{c_{r50}}$ (reference), $\tau^*_{c_{v50}}$ (visual), $\tau^*_{c_{q50}}$ (competence), and $\tau^*_{c_{t50}}$ (theoretical).

Data compiled for each definition of incipient motion are further subdivided by median grain size type (e.g., Table 1a). Values of $\tau^*_{c_{50}}$ have been variously reported in the literature for the median grain size of the surface ($D_{50s}$), subsurface ($D_{50ss}$), and laboratory sediment mixture ($D_{50m}$), the three of which are equal only for uniform-sized sediment; corresponding dimensionless critical shear stresses for these three median grain size types are denoted here as $\tau^*_{c_{50s}}$, $\tau^*_{c_{50ss}}$, and $\tau^*_{c_{50m}}$. Expression of dimensionless critical shear stress in terms of the subsurface grain size distribution was popularized by *Andrews* [1983], who expressed the Shields stress of a given grain size of interest ($\tau^*_{c_i}$) as a power law function of the ratio $D_i/D_{50ss}$; *Andrews* [1983] found that for his data, $D_i/D_{50ss}$ was better correlated with $\tau^*_{c_i}$ than was $D_i/D_{50s}$ ($r^2 = 0.98$ versus 0.89 [*Andrews*, 1983]). Although expression of bed load transport formulations in terms of $D_{50ss}$ [e.g., *Parker et al*., 1982] seems reasonable because of the general correspondence of bed load and subsurface grain size distributions [*Milhous*, 1973; *Kuhnle*, 1993a], it is counterintuitive to use the dimensionless critical shear stress of the subsurface to define thresholds of motion and the onset of bed load transport in gravel-bedded channels [e.g., *Andrews*, 1983; *Parker et al*., 1982]. It is well known that most gravel-bedded rivers are armored and that the surface and subsurface grain size distributions can differ significantly [e.g., *Leopold et al*., 1964; *Milhous*, 1973]. Analysis of incipient motion of gravel-bedded rivers therefore should employ surface values of critical shear stress. No matter how well the subsurface grain size distribution correlates with the bed load transport size distribution, the initiation of bed load transport is controlled by bed surface grains. Nevertheless, the correspondence of subsurface and transport size distributions indicates that subsurface-based mobility values are appropriate for describing bed load transport beyond incipient motion. Because the difference between subsurface and surface grain size distributions is unpredictable, there is no a priori conversion of subsurface-based incipient motion values to surface ones.

There is currently little recognition in the incipient motion literature of the difference between $\tau^*_{c_{50}}$ values for the various $D_{50}$ types (i.e., $\tau^*_{c_{50s}}$, $\tau^*_{c_{50ss}}$, and $\tau^*_{c_{50m}}$). As such, careful evaluation of reported values is necessary in order to compare and choose appropriate values of $\tau^*_{c_{50}}$. For example, using an Andrews-type power function expressed in terms of a generic median grain size, *Komar* [1987a] reports a generic $\tau^*_{c_{50}}$ value of 0.045 for three gravel channels studied by *Milhous* [1973], *Carling* [1983], and *Hammond et al*. [1984]. It is only upon close inspection of *Komar*'s [1987a] analysis that it becomes apparent that this value represents incipient motion of grain sizes similar or equal to the median subsurface size (note 32, appendix); the corresponding unreported surface values ($\tau^*_{c_{50s}}$) range from 0.021 to 0.027 (Table 1c), roughly half that reported by *Komar* [1987a]. Scaling critical shear stresses by subsurface median grain sizes generally produces $\tau^*_{c_{50}}$ values larger than surface-based ones because of bed surface armoring (compare $\tau^*_{c_{50s}}$ and $\tau^*_{c_{50ss}}$ values of *Parker and Klingeman* [1982], *Wilcock and Southard* [1988], *Kuhnle* [1992], *Andrews and Erman* [1986], *Komar* [1987a], and *Komar and Carling* [1991], given in Tables 1a and 1c).

We emphasize that the dimensionless critical shear stress values reported here are for the median grain size only. Shields parameters of other grain sizes of interest will vary as a function of size-specific friction angle, grain protrusion, and mobility of neighboring grains.

## Analysis

Of the 613 dimensionless critical shear stress values compiled in Tables 1a–1e, we examined only those that represent incipient motion of the bed surface, because of their relevance for determining sediment transport thresholds in armored gravel-bedded channels. Subsurface dimensionless critical shear stress values ($\tau^*_{c_{50ss}}$) were removed from the database, as they were all derived from armored channels and thus do not represent initial motion of the streambed surface.

Sorting coefficients ($\sigma_g$) were used to establish conditions in which initial motion of laboratory mixtures could be used as a measure of surface mobility (i.e., establishing when $\tau^*_{c_{50m}}$ approximates $\tau^*_{c_{50s}}$). Poorly sorted laboratory sediment mixtures have the potential to exhibit textural response and reworking prior to measurement of incipient motion in both reference-based and visually based studies. Reference-based laboratory studies commonly employ shear stress and bed load transport data collected after attainment of equilibrium conditions of slope, bed form character, and transport rate [e.g., *Gilbert*, 1914; *Shields*, 1936; *Guy et al*., 1966; *Williams*, 1970; *Wilcock*, 1987], prior to which considerable reworking of the bed surface may occur [e.g., *Wilcock and Southlard*, 1989; *Wilcock and McArdell*, 1993]. Visually based studies also allow varying degrees of water working and sediment transport depending on the specific definition of initial motion employed [e.g., *Kramer*, 1935, *U.S. Waterways Experimental Station* (*USWES*), 1935]. Consequently, the actual surface grain size distribution of initially poorly sorted mixtures may not resemble the original mixture distribution at the time of incipient motion measurement. This causes potentially erroneous results when measured shear stresses for water-worked sediments are combined with unworked mixture distributions to determine dimensionless critical shear stress, as is commonly done in laboratory studies.

Textural response of laboratory mixtures is controlled by relative conditions of transport capacity and sediment supply [e.g., *Dietrich et al*., 1989; *Kinerson*, 1990; *Buffington*. 1995]. Depending on the direction of textural response, $\tau^*_{c_{50m}}$ values could overestimate or underestimate actual dimensionless critical shear stress values of the surface ($\tau^*_{c_{50s}}$). Mixture median grain sizes will approximate surface median grain sizes only when laboratory sediment mixtures are well sorted, as there is little potential for textural response of a well-sorted bed material. Hence only under these conditions will dimensionless critical shear stresses of the mixture approximate those of the surface. We confined our use of mixture-based studies to those using well-sorted material, where "well sorted" is defined as $\sigma_g \leq 0.5$. Under this definition, some of the laboratory sediment mixtures used by *Shields* [1936] are mixed-grain (Table 1a). Mixture-based studies with unknown $\sigma_g$ values were not used.

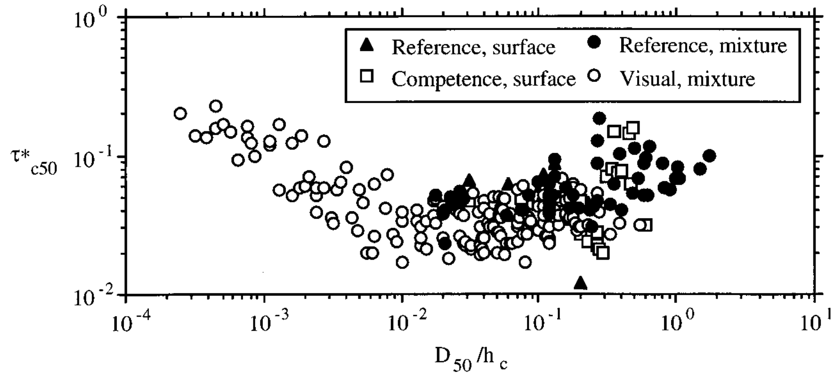


**Figure 2.** Variation of dimensionless critical shear stress with relative roughness. Only data with known $D_{50}/h_c$ are shown here; all mixture data have $\sigma_{gm} \leq 0.5$.

We further screened the data for relative roughness effects ($D_{50}/h_c$). *Bathurst et al.* [1983] demonstrated that for a given grain size, $\tau^*_{c_{50}}$ systematically increases with greater relative roughness and that the rate of increase depends on channel slope [see also *Shields*, 1936; *Cheng*, 1970; *Aksoy*, 1973; *Mizuyama*, 1977; *Torri and Poesen*, 1988]. The increase in $\tau^*_{c_{50}}$ with greater relative roughness can be explained through the concept of shear stress partitioning

$$\tau_0 = \tau' + \tau'' + \cdots \tau^n \tag{1}$$

which is predicated on the hypothesis that the total channel roughness and shear stress ($\tau_0$) can be decomposed into linearly additive components ($\tau'$, $\tau''$, etc.), each characterizing a particular roughness element [*Einstein and Barbarossa*, 1952; *Engelund*, 1966; *Hey*, 1979, 1988; *Parker and Peterson*, 1980; *Brownlie*, 1983; *Prestegaard*, 1983; *Dietrich et al.*, 1984; *Griffiths*, 1989; *Nelson and Smith*, 1989; *Petit*, 1989, 1990; *Robert*, 1990; *Clifford et al.*, 1992; *Millar and Quick*, 1994; *Shields and Gippel*, 1995]. The effective shear stress ($\tau'$) is defined here as that which is available for sediment transport after correction for other roughness elements (i.e., $\tau' = \tau_0 - \tau'' - \cdots \tau^n$).

Based on this concept of shear stress partitioning, greater form drag caused by increased relative roughness ($D_{50}/h_c$) will decrease the shear stress available at the bed for sediment transport ($\tau'$), resulting in a higher total shear stress ($\tau_0$) required for incipient motion and thus an apparently greater $\tau^*_{c_{50}}$ value. The compiled data demonstrate a general positive correlation between $\tau^*_{c_{50}}$ and $D_{50}/h_c$ over the range $0.01 \leq D_{50}/h_c \leq 2$ (Figure 2). The apparent inverse correlation of $\tau^*_{c_{50}}$ and $D_{50}/h_c$ for $D_{50}/h_c < 0.01$ is a coincident effect of flow regime and is not a relative roughness effect. The compiled data with $D_{50}/h_c < 0.01$ generally have $Re^*_c \leq 20$, which corresponds with the hydrodynamically transitional and smooth portions of the *Shields* [1936] curve for which $\tau^*_{c_{50}}$ is negatively correlated with $Re^*_c$ (Figure 1). It is the association with these low $Re^*_c$ flow regimes, not the relative roughness itself, that causes the apparent inverse correlation of $\tau^*_{c_{50}}$ and $D_{50}/h_c$ for $D_{50}/h_c < 0.01$. Because of the influence of relative roughness on $\tau^*_{c_{50}}$, we restricted our analysis to data with $D_{50}/h_c \leq 0.2$, a value that we chose to be generally representative of gravel-bedded streams. We excluded data from studies with unknown $D_{50}/h_c$ values.

We also excluded data from convergent-wall flume studies because of their apparent incompatibility with those from parallel-wall flumes [*Vanoni et al.*, 1966; *Miller et al.*, 1977]. The above screening results in a database of 325 $\tau^*_{c_{50}}$ values, roughly half of the total compilation.

A traditional Shields plot constructed from data representing initial motion of the bed surface exhibits the expected general form of the original Shields curve but reveals systematic methodological biases of incipient motion definition (Plate 1). Values of $\tau^*_{c_{50}}$ determined from reference bed load transport rates ($\tau^*_{c_{r50}}$) and from visual observation of grain motion ($\tau^*_{c_{v50}}$) define subparallel Shields curves, with the visual data generally underlying the reference data. Reference-based dimensionless critical shear stress values determined from well-sorted laboratory mixtures ($\tau^*_{c_{r50m}}$) and from surface grains of natural channels ($\tau^*_{c_{r50s}}$) dovetail quite well. Dimensionless critical shear stress values derived from competence functions ($\tau^*_{c_{q50s}}$) define a separate but poorly developed field. Although not shown, theoretical values ($\tau^*_{c_{t50s}}$) exhibit no trend in relation to $Re^*_c$ and are widely variable depending on choice of intergranular friction angle and grain protrusion (Table 1d). Furthermore, the theoretical values generally predict high stresses that likely represent instantaneous, rather than time-averaged, critical shear stresses [*Buffington et al.*, 1992]. Scatter within Shields curves has long been attributed to methodological differences between experiments [e.g., *Tison*, 1953; *Miller et al.*, 1977; *Carson and Griffiths*, 1985; *Lavelle and Mofjeld*, 1987], but our reanalysis presents the first comprehensive support for this hypothesis.

The data in Plate 1 are also segregated by flow condition (i.e., fully laminar versus hydraulically smooth, transitional, or rough turbulent flow). We did not limit the laminar data to $D_{50}/h_c \leq 0.2$, as relative roughness effects are unlikely for laminar flow conditions. As demonstrated by *Yalin and Karahan* [1979], two Shields curves are defined for laminar versus turbulent flow conditions over similar $Re^*_c$ values. The lower-angle trend of our compiled laminar curve is similar to that identified by *Yalin and Karahan* [1979].

Despite eight decades of incipient motion studies there remains a lack of $\tau^*_{c_{50}}$ values representative of fully turbulent flow and low relative roughness typical of gravel-bedded rivers (Plate 1). The available data indicate that for such conditions reference-based and visually based studies have $\tau^*_{c_{50}}$ ranges of 0.052–0.086 and 0.030–0.073, respectively (Figure 3). The visual range, however, is rather speculative because of the lack of data for high critical boundary Reynolds numbers.

Scatter within the stratified data sets likely reflects a variety of factors, such as differences in bed material properties (i.e.,

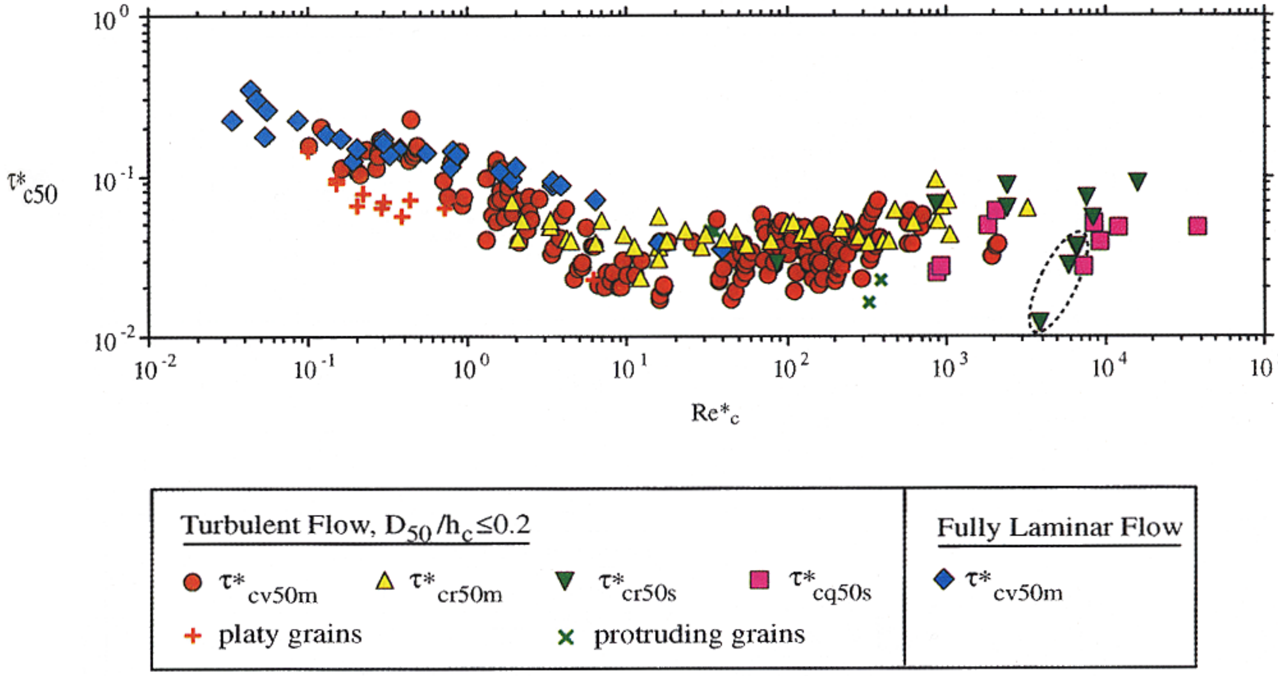


**Plate 1.** Shields curve for empirical data that represent initial motion of the bed surface material. All mixture-based values have known $\sigma_{gm} \leq 0.5$. Circled triangles are values reported for Oak Creek by *Parker and Klingeman* [1982], *Diplas* [1987], *Wilcock and Southard* [1988], *Parker* [1990], and *Wilcock* [1993]; these values are variations of the same data set (that of *Milhous* [1973]) analyzed using *Parker et al.*'s [1982] definition of incipient motion. The reference-based subcategory of protruding grains indicates significant grain projection and exposure sensu *Kirchner et al.* [1990].

grain sorting, packing, shape, and rounding), neglect of roughness elements (i.e., sidewalls, form drag, etc.), method of shear stress measurement, sampling technique used to characterize grain size distributions (and hence $D_{50}$), differences in the scale and duration of sediment transport observations, and finer-scale differences in incipient motion definition [e.g., *Wilcock*, 1988]. Differences in bed material properties can either increase or decrease particle mobility. Greater sorting and angularity cause grains to be more resistant to movement and increase $\tau^*_{c_{50}}$ values [*Shields*, 1936; *Miller and Byrne*, 1966; *Li and Komar*, 1986; *Buffington et al.*, 1992]. In contrast, increased sphericity, looser packing, and surfaces with protruding grains increase grain mobility, resulting in lower $\tau^*_{c_{50}}$ values [*Fenton and Abbott*, 1977; *Church*, 1978; *Reid et al.*, 1985; *Li and Komar*, 1986; *Kirchner et al.*, 1990; *Powell and Ashworth*, 1995]. Incipient motion of nonspherical particles is also influenced by their orientation with respect to the downstream flow direction [*Carling et al.*, 1992]. Platy grains (i.e., slates, micaceous flakes, etc.) tend to have very low incipient motion thresholds (Plate 1 and Figure 3) [*Mantz*, 1977].

Neglecting roughness effects (i.e., use of $\tau_0$ rather than $\tau'$) causes overestimation of $\tau^*_{c_{50}}$ and introduces a range of scatter that varies with the magnitude of neglected roughness. Form drag caused by relative roughness is a source of scatter that is particularly evident in the visually based data. Many of these data cluster into steeply sloping lineaments (Figure 3b). *Bathurst et al.* [1983] explained this observation as a relative roughness effect, with greater relative roughness causing apparently larger $\tau^*_{c_{50}}$ values, as discussed above. Each lineament in Figure 3b is generally composed of data from a single investigation of a particular bed material [*Gilbert*, 1914; *Liu*, 1935; *Wolman and Brush*, 1961; *Neill*, 1967; *Everts*, 1973]. We examined each lineament and found that most demonstrate the expected positive correlation between $D_{50}/h_c$ and $\tau^*_{c_{50}}$; however, *Neill*'s [1967] data inexplicably show a negative correlation despite $Re^*_c$ values greater than 200. Use of the effective shear stress ($\tau'$) rather than the total shear stress ($\tau_0$) in calculating $\tau^*_{c_{50}}$ would likely collapse most of the observed relative roughness lineaments, decreasing $\tau^*_{c_{50}}$.

Although reference-based $\tau^*_{c_{50}}$ values are less variable than visually based ones (Figure 3), potentially large roughness effects caused by bed form drag are commonly neglected in reference studies (Table 1a), which may, in part, explain their larger $\tau^*_{c_{50}}$ values. However, bed form drag also is typically neglected in competence-based studies, which underlie reference-based values (Plate 1). In both reference-based and visually based studies some sidewall corrections are only partial (cf. Tables 1a and 1b). Accurate sidewall correction requires accounting for both differences in bed and wall roughness [e.g., *Vanoni and Brooks*, 1957; *Knight*, 1981; *Flintham and Carling*, 1988; *Shimizu*, 1989] and dissipation of bed shear stress caused by proximity of walls [e.g., *Johnson*, 1942; *Williams*, 1970; *Parker*, 1978; *Knight*, 1981; *Flintham and Carling*, 1988; *Shimizu*, 1989]. Although frequently neglected, the latter correction is of most importance in flume studies, as they typically have narrow width-to-depth ratios (i.e., $W/h < 10$). Partial accounting of sidewall effects can lead to the erroneous conclusion that sidewalls increase the effective bed shear stress [e.g., *Brooks*, 1958; *Wilcock*, 1987; *Kuhnle*, 1993b].

The method used to measure shear stress can also influence $\tau^*_{c_{50}}$. In some instances shear stress is measured as a simple depth-slope product [e.g., *Powell and Ashworth*, 1995], while in other investigations shear stress is estimated from one or more velocity profiles [e.g., *Grass*, 1970; *Ashworth and Ferguson*,

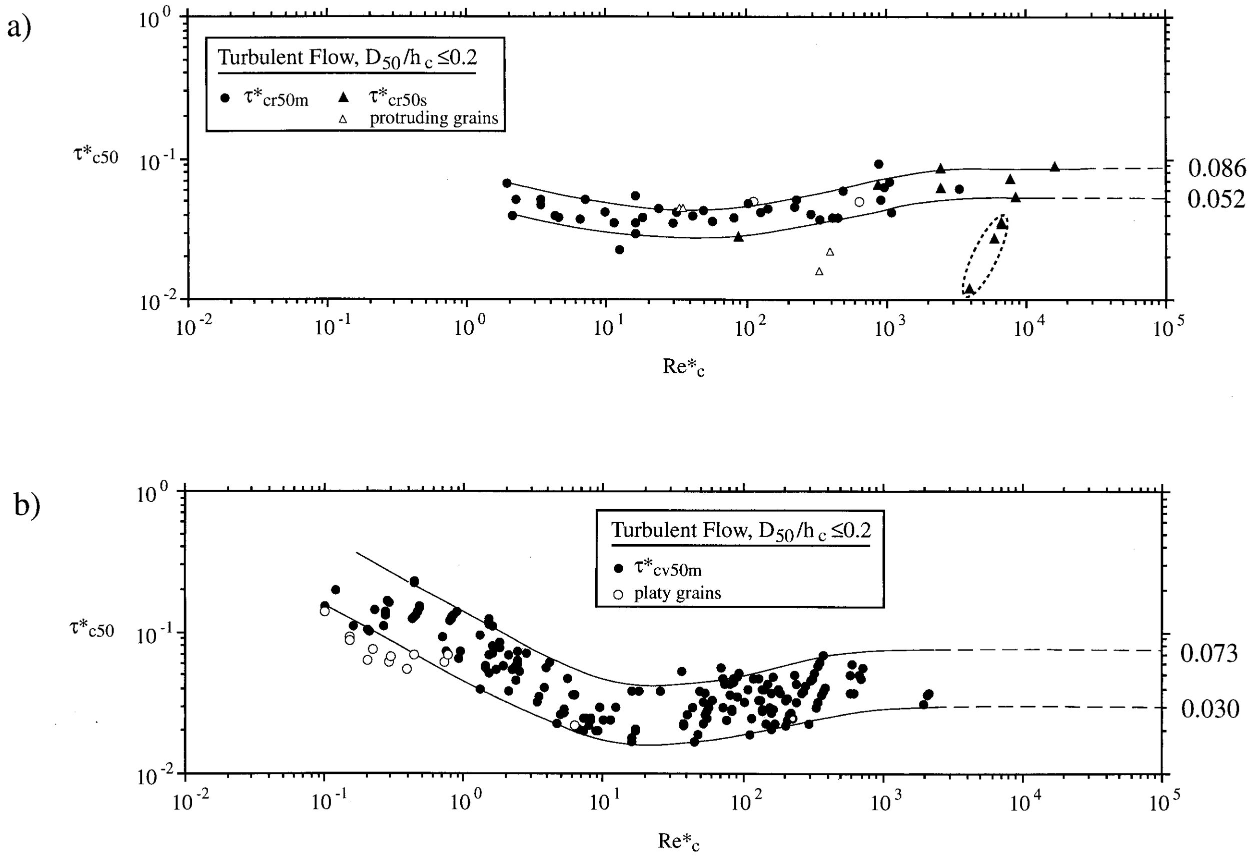


**Figure 3.** Comparison of (a) reference-based and (b) visually based $\tau^*_{c_{50}}$ data shown in Plate 1. Circled triangles in Figure 3a are discussed in caption to Plate 1. See appendix note 12 regarding open circles of Figure 3a.

1989]. Each of these methods can result in different estimates of shear stress, and thus distinct $\tau^*_{c_{50}}$ values, particularly if shear stress is nonuniform through a study reach.

Use of different sampling techniques to characterize grain size distributions, and hence $D_{50}$, may also cause some of the observed scatter. The compiled studies use a variety of areal, grid, and volumetric sampling techniques each of which can yield different results [e.g., *Kellerhals and Bray*, 1971; *Diplas and Sutherland*, 1988; *Diplas and Fripp*, 1992; *Fripp and Diplas*, 1993]. Reference-based studies that use *Parker and Klingeman*'s [1982] method are particularly sensitive to grain size sampling technique, as their method employs the proportion of each size class of the grain size distribution.

Although dimensionless critical shear stress is trigonometrically related to bed slope [e.g., *Wiberg and Smith*, 1987] (a factor not accounted for in the traditional Shields equation), its effect on the compiled $\tau^*_{c_{50}}$ values is minimal, as most of the data are derived from experiments with bed slopes less than 0.01. The data of *Bathurst et al.* [1987] and *Mizuyama* [1977] are notable exceptions; however, $\tau^*_{c_{50}}$ values reported for these studies are based on modified Shields stresses that account for both bed slope and bulk friction angle of the sediment (Table 1a).

Use of appropriate $k_s$ values when calculating $Re^*_c$ may reduce some of the observed scatter [e.g., *Ippen and Verma*, 1953]. There have been numerous $k_s$ empiricisms proposed [cf. *Einstein and Barbarossa*, 1952; *Leopold et al.*, 1964; *Kamphuis*, 1974; *Hey*, 1979; *Bray*, 1980], most of which are greater than $D_{50s}$ for heterogeneous bed surfaces; *Whiting and Dietrich* [1990], for example, suggest $k_s = 3D_{84s}$. Although we use $k_s = D_{50s}$ for comparison with historical Shields curves, $k_s = D_{50s}$ is only appropriate for uniformly sized sediment. However, $Re^*_c$ correction using appropriate $k_s$ values will not improve the $\tau^*_{c_{50}}$ uncertainty, which accounts for most of the observed scatter.

Differences in the scale and duration of observation within and between methodologies may also contribute to the scatter of compiled data [e.g., *Neill and Yalin*, 1969; *Fenton and Abbott*, 1977; *Wilcock*, 1988]. For example, the spatial scale of observation in visually based studies varies from the entire bed surface [e.g., *Gilbert*, 1914] to that viewed from a microscope [*White*, 1970]. Similarly, reference- and competence-based studies may employ channel-spanning bed load traps that sample all material passing a cross section [e.g., *Milhous*, 1973] or they may combine several point measures of bed load transport [e.g., *Ashworth and Ferguson*, 1989], representing a smaller scale of observation. Temporal scales of observation also vary among and within methodologies. For example, visually based studies are typically of short duration and made while the channel adjusts to perturbations of slope or hydraulic discharge. In contrast, reference-based studies conducted in flumes employ data collected over long time periods and after attainment of equilibrium conditions of channel morphology and hydraulics. However, reference- and competence-based studies conducted in the field are influenced by nonequilibrium conditions and may require shorter periods of data collection because of logistics and safety during high flows [e.g., *Ashworth and Ferguson*, 1989; *Wilcock et al*., 1996]. Differences in spatial and temporal scales of observation can yield different estimates of critical conditions for incipient motion, particularly in channels that exhibit nonuniformities of shear stress, grain size, and bed material properties that cause spatial differences in mobility [e.g., *Powell and Ashworth*, 1995; *Wilcock et al*., 1996]. Rules for standardizing incipient motion definition and the spatial and temporal scales of observation between investigations have been proposed [e.g., *Neill and Yalin*, 1969; *Yalin*, 1977; *Wilcock*, 1988] but are not widely used. *Wilcock* [1988] proposed a standard definition of incipient motion for mixed-grain sediments that accounts for the number of grains moved, their size and proportion of the grain size distribution, and the area and duration of observation. Even when such rules are applied, channels with identical reach-average shear stresses and grain size distributions may demonstrate different $\tau^*_{c_{50}}$ values because of subreach differences in the spatial variability of shear stress, sediment supply, and bed surface textures (i.e., grain size, sorting, and packing).

Differences of incipient motion definition within each methodology may also contribute to the observed scatter. For example, *Kramer*'s [1935] three definitions of visual grain motion (weak, medium, and general) represent a two-fold difference in $\tau^*_{c_{50}}$ values. Similarly, differences in the choice of dimensionless reference transport rate used to define incipient motion can result in a three-fold variation of reference-based $\tau^*_{c_{50}}$ values [*Paintal*, 1971, Figure 6]. Despite this potential for variation, *Wilcock* [1988] found that reference-based $\tau^*_{c_{50}}$ values determined from the *Parker and Klingeman* [1982] and *Ackers and White* [1973] methods differed by only 5% for the same data set.

Scatter within the reference- and competence-based data may also reflect choice of curve fitting technique [*Diplas*, 1987; *Ashworth and Ferguson*, 1989; *Ashworth et al*., 1992; *Wathen et al*., 1995]. In an extreme example, *Paintal* [1971, Figure 8] demonstrates that nonlinear relationships between bed load transport rate and dimensionless shear stress that are mistakenly fit with a linear function can cause up to a five-fold overestimation of reference-based $\tau^*_{c50}$ values. In many cases it is difficult to assess or correct differences in curve-fitting technique between investigations due to incomplete documentation of measurements and analysis procedure. The results of *Shields* [1936], in particular, are often used as the standard for comparison, yet *Shields*' [1936] basic measurements and curve fitting technique are unreported, making it difficult to fully assess the causes of discrepancy between *Shields*' [1936] data and those reported by others.

The above influences on $\tau^*_{c_{50}}$ values can be of comparable magnitude and can easily account for the observed scatter of values within methodologies. For example, neglect of roughness effects and natural variation of bed surface characteristics have the potential to cause similar magnitudes of scatter. Variation in particle protrusion and packing can result in an order of magnitude range in $\tau^*_{c_{50}}$ [*Fenton and Abbott*, 1977; *Powell and Ashworth*, 1995], while bed form drag in natural rivers can comprise 10%–75% of the total channel roughness [*Parker and Peterson*, 1980; *Prestegaard*, 1983; *Dietrich et al*., 1984; *Hey* 1988], indicating a similar range of $\tau^*_{c_{50}}$ variation if bed form resistance is not accounted for. Nonetheless, despite potentially similar sources and/or magnitudes of scatter, the compiled data demonstrate distinct methodological biases (Plate 1).

## Discussion

Our reanalysis and stratification of incipient motion values reveal systematic methodological biases and highlight fundamental differences of median grain size type and their associated values of dimensionless critical shear stress. The Shields curve constructed from our data compilation (Plate 1) (1) specifically represents incipient motion of the bed surface, (2) excludes data associated with large relative roughness values that are uncharacteristic of gravel-bedded rivers, and (3) includes for the first time reference- and competence-based $\tau^*_{c_{50}}$ values for surface material ($\tau^*_{c_{r50s}}$ and $\tau^*_{c_{q50s}}$). We find that the rough, turbulent flow value of $\tau^*_{c_{50}} \approx 0.045$ reported in previous compilation studies [*Miller et al*., 1977; *Yalin and Karahan*, 1979] is typical of visually determined mobility thresholds of laboratory mixtures ($\tau^*_{c_{v50m}}$), but underestimates dimensionless critical shear stresses determined from reference transport rates ($\tau^*_{c_{r50m}}$ and $\tau^*_{c_{r50s}}$) (Figure 3).

Although methodological bias explains much of the scatter in our constructed Shields curve, one is still faced with deciding which investigative method to rely on when choosing a $\tau^*_{c_{50}}$ value. None of the four investigative methods is demonstrably superior; each has its strengths and weaknesses. However, some methods may be more appropriate for particular applications [*Carson and Griffiths*, 1985]. For example, because reference- and competence-based values are derived from bed load transport measures, they may be more well suited to application in bed load transport investigations. Depending on the bed load sampling strategy, reference- and competence-based methods may also integrate differential bed mobility resulting from bed surface textural patches and reach-scale divergence of shear stress and sediment supply and thus may be more appropriate for representing reach-average bed mobility. In contrast, visually based methods typically record local incipient motion and are best applied to mobility studies of discrete bed surface textural patches. Because of methodological biases, care should be taken to choose $\tau^*_{c_{50}}$ values from an investigative method that represents the scale and type of incipient motion needed for one's particular study goals. Conversely, study results should be interpreted in light of the incipient motion method used and the sediment transport processes that it measures. For example, $\tau^*_{c_{50}}$ values from either competence- or reference-based methods could be used to predict reach-average incipient motion, but competence-based values describe motion of the coarsest bed load sizes, whereas reference-based values describe motion of the full bed load distribution; the two methods describe the mobility of different subpopulations of the bed material and may yield different results if equivalent scaling factors are not used [*Wilcock*, 1988].

Of the four methods from which to choose $\tau^*_{c_{50}}$ values, competence-based and theoretically based methods can be excluded because of a paucity of data that precludes confident interpretation of the functional relationship between $\tau^*_{c_{50}}$ and $Re^*_c$. Nevertheless, the competence-based data define a horizontal band of roughly constant dimensionless critical shear stresses at high $Re^*_c$ values as expected for a Shields-type relationship (Plate 1). Furthermore, this band of data generally lies within the Shields curve defined by visually based methods and systematically underlies the reference-based data (cf. Plate 1 and Figure 3), contrary to expectations that competence values should be greater than reference-based ones due to underrepresentation of coarse grain sizes [*Wilcock*, 1992b].

The fact that some of the data in both methods are derived from the same study sites [*Milhous*, 1973; *Ashworth et al*., 1992; *Wathen et al*., 1995] makes this difference between competence- and reference-based approaches credible. For the same study site, competence-based $\tau^*_{c_{50}}$ values are roughly 15%–30% smaller. The systematically lower incipient motion values determined from the competence approach may reflect an inherent bias associated with use of the largest mobile grain size. Larger bed surface grains may have lower mobility thresholds because of greater protrusion and smaller intergranular friction angles [e.g., *Buffington et al*., 1992]. *Komar and Carling*'s [1991] variant of the competence approach using the median grain size of the load, rather than the maximum grain size, produces $\tau^*_{c_{50}}$ values similar to reference-based ones (Table 1e).

In contrast to theoretically based and competence-based methods, functional relationships between $\tau^*_{c_{50}}$ and $Re^*_c$ are well defined for reference-based and visually based approaches. Both the reference-based and visually based studies exhibit a roughly twofold range in $\tau^*_{c_{50}}$ values for conditions typical of gravel-bedded channels (Figure 3), which represents significant uncertainty in dimensionless critical shear stress. Many bed load transport equations are based on the difference between the applied and critical grain shear stresses raised to some power greater than 1 (see the review by *Gomez and Church* [1989]). Differences between applied and critical shear stresses are typically small in gravel-bedded channels [*Parker et al*., 1982] because of the approximately bankfull-threshold nature of bed mobility (see the review by *Buffington* [1995]). Consequently, small errors in $\tau_{c_{50}}$ due to uncertainty in $\tau^*_{c_{50}}$ can cause significant errors in calculated bed load transport rates.

Consideration of the sources of scatter and their systematic influence on the reference-based and visually based data provides further guidance in choosing specific $\tau^*_{c_{50}}$ values. In particular, neglect of form drag effects may cause systematic overestimation of $\tau^*_{c_{50}}$ values. It is commonly implied that because flume-based studies of incipient motion employ initially planar bed surfaces they are free of form drag influences caused by bed forms [e.g., *Miller et al*., 1977]. This is true for the visually based studies, but it is not so for most of the reference-based investigations, such as *Shields*' [1936]. In the visual studies, flow is typically increased gradually until grains are observed to move from a plane-bed surface [e.g., *Kramer*, 1935; *White*, 1970; *Yalin and Karahan*, 1979]. In contrast, most of the reference studies are based on bed load transport data collected after attainment of equilibrium conditions, which in many instances are characterized by the presence of bed forms [e.g., *Gilbert*, 1914; *Shields*, 1936; *Guy et al*., 1966; *Wilcock and Southard*, 1988]. Because bed form resistance can comprise up to 75% of the total channel roughness [*Hey*, 1988], there is a potentially significant difference between $\tau'$ and $\tau_0$, and hence the calculated $\tau^*_{c_{50}}$ value, if bed form roughness is not accounted for. Moreover, it is uncertain whether bed load transport data from surfaces characterized by bed forms can provide a meaningful extrapolation to conditions of initial motion from a lower-regime plane bed, as is commonly intended in laboratory reference-based studies. Although bed form resistance is not an issue in visually based studies, relative roughness effects common to these studies (i.e., lineaments of Figure 3b) may provide an equally important source of form drag and overestimation of $\tau^*_{c_{50}}$ values if not accounted for.

In analyzing incipient motion data it is common practice to fit a single average curve through the scatter of data. However, a Shields curve defined by minimum $\tau^*_{c_{50}}$ values will minimize overestimation of $\tau^*_{c_{50}}$ caused by neglect of form drag resistance in both reference-based and visually based studies (Tables 1a and 1b) and may be more representative of poorly sorted sediments typical of gravel channels. Poorly sorted sediments tend to have lower intergranular friction angles and thus lower incipient motion thresholds [*Buffington et al*., 1992]. The necessarily narrow range of sorting ($\sigma_g \leq 0.5$) of the mixture data, however, may preclude any meaningful analysis of sorting effects. The $\tau^*_{c_{50}}$ values for rough turbulent flow derived from minimum Shields curves are 0.052 and 0.030 for reference-based and visually based studies, respectively (Figure 3). However, thorough accounting of roughness effects may produce even lower values.

Regardless of whether an average or minimum curve is chosen for the reference-based data, Oak Creek is an outlier (Figure 3a). It does not fit with the expected general form of the traditional Shields curve as defined by the other data. This is somewhat disconcerting, as Oak Creek is believed to be one of the best bed load transport data sets available for natural channels and has been used by many authors as the standard for comparison. Although the issue warrants further investigation, the discrepancy between Oak Creek and the other reference-based data may be due to unaccounted for differences in channel roughness (Table 1a).

## Conclusions

Our reanalysis of incipient motion data for bed surface material indicates that (1) much of the scatter in Shields curves is due to systematic biases that investigators should be aware of when choosing and comparing dimensionless critical shear stress values from the literature; and (2) there is no definitive $\tau^*_{c_{50}}$ value for the rough, turbulent flow characteristic of gravel-bedded rivers, but rather there is a range of values that differs between investigative methodologies. Our analysis indicates that less emphasis should be placed on choosing a universal $\tau^*_{c_{50}}$ value, while more emphasis should be placed on choosing defendable values for particular applications, given the observed methodological biases, uses of each approach, and systematic influences of sources of uncertainty associated with different methods and investigative conditions.

**Note added in proof.** During the time this article was in press we discovered several other referenced-based values similar to those of Oak Creek [see *Andrews*, 1994; *Andrews and Nankervis*, 1995].

## Notation

| | |
|---|---|
| $D_{50}$, $D_{84}$, $D_i$ | grain size for which 50%, 84%, and $i$% of the grains are finer. |
| $D_{50s}$, $D_{50ss}$, $D_{50m}$, $D_{50l}$ | median grain sizes of the surface, subsurface, laboratory mixture, and bed load. |
| $D_{gs}$ | geometric mean surface grain size. |
| $g$ | gravitational constant. |
| $h_c$ | critical flow depth for incipient motion. |
| $k_s$ | boundary roughness length scale (equivalent to |

**Table 1a.** Previously Reported $\tau^*_{c_{50}}$ Values: Reference Transport Rate

| Source | Note* | $\tau^*_{c_{r50s}}$ | $Re^*_c$† | Proposed $\tau^*_c$ Function Other Than Shields' | $D_{50s}$, mm | $\sigma_{gs}$ ($\phi$) | $\rho_s$, kg/m$^3$ | $D_{50s}/h_c$‡ | Experimental Conditions |
|---|---|---|---|---|---|---|---|---|---|
| | | | | *Parker and Klingeman [1982] Reference Transport Rate* | | | | | |
| *Parker and Klingeman* [1982] | 1 | 0.035§ | 6,744 | $\tau^*_{c_{ri}} = 0.035(D_i/D_{50s})^{-0.94}$ | 54 | 1.09 | 2850 | 0.15 | natural pool-riffle channel (Oak Creek); no form drag or sidewall correction |
| *Diplas* [1987] | | 0.034§ | 6,647 | $\tau^*_{c_{ri}} = 0.087(D_i/D_{50ss})^{-0.94}$ | 54 | 1.09 | 2850 | 0.16 | same as *Parker and Klingeman* [1982] |
| From *Wilcock and Southard* [1988] | 2 | | | | | | | | |
| *Milhous* [1973] | | 0.027§ | 5,923 | $\tau^*_{c_{ri}} = 0.073(D_i/D_{50ss})^{-0.98}$ | 54 | 1.09 | 2850 | 0.20 | same as *Parker and Klingeman* [1982] |
| *Ashworth and Ferguson* [1989] | 3 | 0.072§ | ~7,773 | $\tau^*_{c_{ri}} = 0.072(D_i/D_{50s})^{-0.65}$ | ~50 | m | 2540 | 0.11 | natural pool-riffle channel (Alt Dubhaig), variable sinuosity; sidewall correction implicit |
| | | 0.054§ | ~8,463 | $\tau^*_{c_{ri}} = 0.054(D_i/D_{50s})^{-0.67}$ | ~57.5 | m | 2600 | 0.10 | natural pool-riffle channel (River Feshie), mildly braided; sidewall correction implicit |
| | | 0.087§ | ~16,138 | $\tau^*_{c_{ri}} = 0.087(D_i/D_{50s})^{-0.92}$ | ~69 | m | 3090 | 0.13 | natural braided channel (Lyngsdalselva); sidewall correction implicit |
| *Parker* [1990] | | 0.034§ | 6,731 | $\tau^*_{c_{ri}} = 0.039(D_i/D_{gs})^{-0.90}$ | 54 | 1.02 | 2850 | 0.16 | same as *Parker and Klingeman* [1982] |
| *Ashworth et al.* [1992] | | 0.061§ | 2,463 | $\tau^*_{c_{ri}} = 0.061(D_i/D_{50s})^{-0.79}$ | 24 | m | 2650 | 0.06 | natural, braided, gravel channel (Sunwapta River); form drag and sidewall correction as in note 3 |
| *Kuhnle* [1992] | | 0.065§ | 869 | $\tau^*_{c_{ri}} = 0.086(D_i/D_{50ss})^{-0.81}$ | 11.73 | 0.90 | ··· | 0.03 | natural channel with mixed gravel and sand bed exhibiting macroscale dunes [*Kuhnle and Bowie*, 1992] (Goodwin Creek); no sidewall or form drag correction |
| From *Wilcock* [1993] | 4 | | | | | | | | |
| *Milhous* [1973] | | 0.012§ | 3,949 | $\tau^*_{c_{ri}} = 0.033(D_i/D_{50ss})^{-0.98}$ | 54 | 1.09 | 2850 | 0.20 | same as *Parker and Klingeman* [1982]; *Einstein* [1950] sidewall and form drag correction¶ |
| *Wilcock and McArdell* [1993] | 5 | 0.028§ | 88 | $\tau^*_{c_{ri}} = 0.028(D_i/D_{50s})^{-0.45}$, $0.77 \leq D_i/D_{50s} \leq 17.3$ | 2.6 | 2.51 | 2610 | 0.12 | straight, rectangular flume; variable bedforms; *Einstein* [1950] sidewall and form drag correction¶ |
| *Wathen et al.* [1995] | | 0.086§ | 2,445 | $\tau^*_{c_{ri}} = 0.086(D_i/D_{50s})^{-0.90}$ | 21.3 | ~1.6 | 2650 | 0.01–0.14 | natural pool-riffle channel (Alt Dubhaig), variable sinuosity; no sidewall or form drag correction |
| | | | | *Ackers and White [1973] Reference Transport Rate* | | | | | |
| *Day* [1981] | 6 | 0.035 | 2,129 | ··· | 20 | 0.92 | ··· | ≥0.24–0.41 | straight, rectangular flume; plane bed; no sidewall correction |
| | | 0.036 | 2,491 | ··· | 22 | 0.86 | ··· | ≥0.28–0.45 | as above |
| | | | | *Zero Reference Transport* | | | | | |
| *Ippen and Verma* [1953] | | 0.045 | 30 | ··· | 2.0 | ≤0.13 | 1280 | 0.23 | straight, rectangular flume; plastic test grain placed on fixed plane bed; significant grain projection and exposure;** sidewall effects insignificant |

**Table 1a.** (Continued)

| Source | Note* | Surface Grain Size Distribution: $\tau^*_{c_{r50s}}$ | $Re^*_c$† | Proposed $\tau^*_c$ Function Other Than Shields' | $D_{50s}$, mm | $\sigma_{gs}$ ($\phi$) | $\rho_s$, kg/m$^3$ | $D_{50s}/h_c$‡ | Experimental Conditions |
|---|---|---|---|---|---|---|---|---|---|
| | | 0.044§ | 33 | ··· | 2.0 | ≤0.13 | 1280 | 0.19 | as above |
| | | 0.045§ | 35 | ··· | 2.0 | ≤0.13 | 1280 | 0.16 | as above |
| *Meland and Norrman* [1966] | 7 | 0.008§ | 33 | ··· | 2.09 | 0 | 2560 | ≥0.05 | straight, rectangular flume; fixed plane bed; glass test grain placed on rhombohedrally packed bed; significant grain projection and exposure;** sidewall correction implicit |
| | | 0.022§ | 390 | ··· | 7.76 | 0 | 2510 | ≥0.16 | as above |
| | | 0.016§ | 330 | ··· | 7.76 | 0 | 2510 | ≥0.16 | as above, but with a loose bed |

| Source | Note* | Laboratory Mixture Grain Size Distribution: $\tau^*_{c_{r50m}}$ | $Re^*_c$† | Proposed $\tau^*_c$ Function Other Than Shields' | $D_{50m}$, mm | $\sigma_{gm}$ ($\phi$) | $\rho_s$, kg/m$^3$ | $D_{50m}/h_c$‡ | Experimental Conditions |
|---|---|---|---|---|---|---|---|---|---|
| | | | | *Zero Reference Transport Rate* | | | | | |
| From *Shields* [1936] | 8 | | | | | | | | |
| *Gilbert* [1914]†† | | 0.059§ | 489 | ··· | 7.01 | <0.22 | 2690? | 0.16 | straight, rectangular flume; bed form types not reported; partial sidewall correction (?);‡‡ no form drag correction; subrounded grains |
| | | 0.051§ | 227 | ··· | 4.94? | <0.26 | 2690? | 0.18 | as above |
| *Casey* [1935] | | 0.067§ | 1.9 | ··· | 0.17 | 0.41 | 2650 | ≤0.03 | straight, rectangular flume; various bed forms; partial sidewall correction (?);‡‡ no form drag correction; subangular to rounded grains |
| | | 0.052§ | 3.3 | ··· | 0.27 | 0.42 | 2650 | ≤0.04 | as above |
| | | 0.035§ | 11 | ··· | 0.68 | 0.16 | 2650 | ≤0.10 | as above |
| | | 0.035§ | 16 | ··· | 0.87 | 0.17 | 2650 | ≤0.12 | as above |
| | | 0.038§ | 18 | ··· | 0.94 | 0.19 | 2650 | ≤0.05 | as above |
| | | 0.042§ | 31 | ··· | 1.26 | 0.20 | 2650 | <0.20? | as above |
| | | 0.043§ | 49 | ··· | 1.75 | 0.19 | 2650 | ≤0.10 | as above |
| | | 0.039§ | 80 | ··· | 2.46 | 0.22 | 2650 | ≤0.10 | as above |
| *Kramer* [1935] | | 0.033 | 6.6 | ··· | 0.53 | 0.81 | 2700? | ≤0.04 | straight, rectangular flume; various bed forms; partial sidewall correction (?);‡‡ no form drag correction; well-rounded grains |
| | | 0.039 | 7.6 | ··· | 0.51 | 0.74 | 2700? | ≤0.04 | as above |
| | | 0.033 | 7.8 | ··· | 0.55 | 0.62 | 2700? | ≤0.04 | as above |
| *USWES* [1935]§§? | | 0.051§ | 2.2 | ··· | 0.21? | 0.32 | 2650 | 0.02 | straight, rectangular flume; various bed forms; partial sidewall correction (?);‡‡ no form drag correction |
| | | 0.046 | 4.1 | ··· | 0.31? | 0.53? | 2650 | 0.02 | as above |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 0.039§ | 4.4 | ··· | 0.35? | 0.37? | 2650 | 0.02 | as above |
| | | 0.035 | 7.4 | ··· | 0.52? | 0.53? | 2650 | 0.03 | as above |
| | | 0.036 | 7.4 | ··· | 0.51? | 0.82? | 2650 | 0.03 | as above |
| | | 0.036 | 7.8 | ··· | 0.52? | 0.53? | 2650 | 0.03 | as above |
| *Shields* [1936] | | 0.037§ | 6.3 | ··· | 0.36 | 0.30 | 4300 | ≤0.04? | straight, rectangular flume; various bed form types; form drag and sidewall correction as in note 3; angular barite grains |
| | | 0.036§ | 56 | ··· | 1.52 | 0.35 | 4200 | ≤0.04? | as above |
| | | 0.042§ | 124 | ··· | 2.46? | 0.22 | 4190 | ≤0.04? | as above |
| | | 0.046§ | 219 | ··· | 3.44 | 0.16 | 4200 | ≤0.04? | as above |
| | | 0.044§ | 142 | ··· | 2.76? | 0.41 | 4200 | ≤0.04? | as above |
| | | 0.038 | 8.8 | ··· | 1.56 | 0.59 | 1060 | ≤0.04? | as above, but with angular amber grains; partial sidewall correction;‡‡ no form drag correction |
| | | 0.041 | 8.9 | ··· | 1.56 | 0.59 | 1060 | ≤0.04? | as above |
| | | 0.045 | 9.3 | ··· | 1.56 | 0.59 | 1060 | ≤0.04? | as above |
| | | 0.030§ | 16 | ··· | 0.85 | 0.23 | 2700 | ≤0.04? | as above, but with angular granitic grains |
| | | 0.035§ | 29 | ··· | 1.23 | 0.23 | 2710 | ≤0.04? | as above |
| | | 0.049§ | 100 | ··· | 2.44 | 0.23 | 2690 | ≤0.04? | as above |
| | | 0.030 | 15 | ··· | 1.77? | 0.78? | 1270 | ≤0.04? | as above, but with angular coal grains |
| | | 0.038 | 20 | ··· | 1.77 | 0.78 | 1270 | ≤0.04? | as above |
| | | 0.040 | 23 | ··· | 1.88 | 0.72 | 1270 | ≤0.04? | as above |
| | | 0.049 | 38 | ··· | 2.53 | 0.56 | 1270 | ≤0.04? | as above |
| From *Johnson* [1943] | 9 | | | | | | | | |
| *Gilbert* [1914]†† | 10 | 0.297 | 12 | ··· | 0.305 | <0.09 | 2690 | 0.02 | straight, rectangular flume; various bed forms; *Einstein* [1942] sidewall correction;¶ no form drag correction; subangular grains |
| | | 0.262 | 15 | ··· | 0.375 | <0.11 | 2690 | 0.02 | as above |
| *MacDougall* [1933]; River Hydraulics Laboratory, (unpublished report) | | 0.055§ | 16 | ··· | 0.67 | 0.38 | ··· | 0.03 | straight, rectangular flume; bed form types not reported; sidewall correction implicit (?); rounded grains |
| *Chyn* [1935] | | 0.062 | 14 | ··· | 0.60 | 1.23 | ··· | 0.02 | as above, but with plane-bed or dune morphology; no form drag correction |
| | | 0.044§ | 23 | ··· | 0.83 | 0.30 | ··· | 0.02 | as above, but with dunes only |
| *Jorissen* [1938] | | 0.064 | 16 | ··· | 0.62 | 0.85 | 2670 | 0.02 | as above, but with planar or rippled bed; no form drag correction |
| | | 0.044 | 24 | ··· | 0.92 | 0.61 | 2670 | 0.03 | as above |
| *Meland and Norrman* [1969] | 11 | 0.047 | 207 | ··· | 3.9 | ··· | 2560 | <0.08? | straight, rectangular flume; various bed forms; form drag and sidewall correction same as in note 3; glass grains |
| *Paintal* [1971] | 12 | 0.050§ | 638 | ··· | 7.95 | 0.18 | 2650 | 0.12 | straight, rectangular flume; plane bed (?); *Johnson* [1942] sidewall correction¶ |
| | | 0.050§ | 112 | ··· | 2.5 | 0.14 | 2650 | 0.02 | as above |
| *Sternberg* [1971]§§,‖‖ | | 0.048 | 4.8 | ··· | 0.42 | 1.09 | ··· | $<2 \times 10^{-5}$ | sandy marine channel (Pickering Passage) with rippled bed; roughness correction similar to that for note 3 |
| *Mizuyama* [1977]††,¶¶ | 13 | 0.038§ (0.049) | 437 | $\tau^*_{c_{r50m}} = \tau_{c_{r50m}}/[(\rho_s - \rho) g D_{50m} (\tan\Phi \cos\theta - \sin\theta)]$ | 6.4 | <0.12 | 2507 | 0.12 | straight, rectangular flume; bed form type not reported; *Einstein* [1942] sidewall correction;¶ no form drag correction |
| | | 0.047 (0.060) | 481 | as above | 6.4 | <0.12 | 2507 | 0.27 | as above |
| | | 0.053 (0.066) | 507 | as above | 6.4 | <0.12 | 2507 | 0.49 | as above |
| | | 0.042§ (0.041) | 1,076 | as above | 12.0 | <0.21 | 2656 | 0.12 | as above |
| | | 0.044 (0.043) | 1,096 | as above | 12.0 | <0.21 | 2656 | 0.33 | as above |

**Table 1a.** (Continued)

| Source | Note* | $\tau^*_{c_{r50m}}$ | $Re^*_c$† | Proposed $\tau^*_c$ Function Other Than Shields' | $D_{50m}$, mm | $\sigma_{gm}$ ($\phi$) | $\rho_s$, kg/m$^3$ | $D_{50m}/h_c$‡ | Experimental Conditions |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | Laboratory Mixture Grain Size Distribution | | | | |
| | | 0.052(0.049) | 1,174 | as above | 12.0 | <0.21 | 2656 | 0.59 | as above |
| | | 0.058 (0.054) | 1,227 | as above | 12.0 | <0.21 | 2656 | 0.83 | as above |
| | | 0.067 (0.060) | 1,297 | as above | 12.0 | <0.21 | 2656 | 1.00 | as above |
| | | 0.083 (0.072) | 1,419 | as above | 12.0 | <0.21 | 2656 | 1.04 | as above |
| | | 0.043 (0.042) | 2,637 | as above | 22.5 | <0.08 | 2490 | 0.25 | as above |
| | | 0.051 (0.048) | 2,841 | as above | 22.5 | <0.08 | 2490 | 0.63 | as above |
| | | 0.057 (0.053) | 2,971 | as above | 22.5 | <0.08 | 2490 | 0.90 | as above |
| | | 0.069 (0.061) | 3,199 | as above | 22.5 | <0.08 | 2490 | 1.05 | as above |
| | | 0.079 (0.066) | 3,320 | as above | 22.5 | <0.08 | 2490 | 1.47 | as above |
| | | 0.098 (0.077) | 3,580 | as above | 22.5 | <0.08 | 2490 | 1.73 | as above |
| *Pazis and Graf* [1977] | 14 | ~0.020 | 6.2–92 | ⋯ | 0.49–3.02 | ~u? | 2650–1410 | <0.02 | straight, rectangular flume; sand and plastic grains; bed form types unreported; *Einstein* [1950] sidewall and form drag correction (?)¶ |
| From *Bathurst et al.* [1987] | 15 | | | | | | | | |
| École Polytechnique Fédérale de Lausanne | | 0.052§ (0.036) | 905 | $\tau^*_{c_{r50m}} = \tau_{c_{r50m}}/[(\rho_s - \rho)gD_{50m}(\tan\Phi\cos\theta - \sin\theta)]$ | 11.5 | ~u? | 2650 | 0.08 | straight, rectangular flume; various bed forms; *Einstein* [1950] (?) sidewall and form drag (?) correction¶ |
| | | 0.063§ (0.043) | 944 | as above | 11.5 | ~u? | 2650 | 0.10 | as above |
| | | 0.070§(0.048) | 1,036 | as above | 11.5 | ~u? | 2650 | 0.13 | as above |
| | | 0.062§ (0.053) | 3,231 | as above | 22.2 | 0.34 | 2570 | 0.12 | as above |
| | | 0.087 (0.071) | 3,436 | as above | 22.2 | 0.34 | 2570 | 0.27 | as above |
| | | 0.102 (0.082) | 3,621 | as above | 22.2 | 0.34 | 2570 | 0.39 | as above |
| | | 0.113 (0.088) | 3,753 | as above | 22.2 | 0.34 | 2570 | 0.50 | as above |
| | | 0.115 (0.088) | 3,886 | as above | 22.2 | 0.34 | 2570 | 0.65 | as above |
| | | 0.061 (0.049) | 7,976 | as above | 44.3 | ~u? | 2750 | 0.35 | as above |
| | | 0.068 (0.054) | 8,366 | as above | 44.3 | ~u? | 2750 | 0.53 | as above |
| | | 0.088 (0.068) | 8,714 | as above | 44.3 | ~u? | 2750 | 0.59 | as above |
| | | 0.087 (0.065) | 9,131 | as above | 44.3 | ~u? | 2750 | 0.79 | as above |
| *Bathurst et al.* [1979] | 16 | 0.094§ (0.092) | 881 | as above | 8.8 | 0.42 | 2629 | 0.13 | straight, rectangular flume; plane-bed or low amplitude bed forms; no form drag or sidewall correction (?) |
| | | 0.126 (0.113) | 974 | as above | 8.8 | 0.42 | 2629 | 0.27 | as above |
| | | 0.185 (0.170) | 1,194 | as above | 8.8 | 0.42 | 2629 | 0.28 | as above |
| | | 0.095(0.079) | 6,198 | as above | 34 | 0.44 | 2629 | 0.61 | as above |
| *Li and Komar* [1992] | 17 | 0.048§ | 3.3 | ⋯ | 0.24 | ≤0.13 | ⋯ | 0.03 | straight, rectangular flume; nonuniform flow; bed form types unreported; no form drag or sidewall correction |
| | | | | *Ackers and White [1973] Reference Transport Rate* | | | | | |
| From *Day* [1980] | | | | | | | | | |
| *USWES* [1935] | 18 | 0.050 | 9.1 | ⋯ | 0.42 | 0.86 | 2650 | 0.02 | straight, rectangular flume; various bed forms; no sidewall or form drag correction; subangular to subrounded grains |
| | | 0.047 | 7.3 | ⋯ | 0.44 | 0.59 | 2650 | 0.02 | as above, but with angular to subrounded grains |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 0.034 | 189 | ··· | 4.10 | 0.54 | 2650 | 0.07 | as above, but with subrounded to subangular grains |
| *Day* [1980] | | 0.024 | 37 | ··· | 1.75 | 2.10 | ··· | 0.04 | straight, rectangular flume; bed form type not reported; no sidewall or form drag correction |
| | | 0.029 | 34 | ··· | 1.55 | 1.70 | ··· | 0.03 | as above |
| *Day* [1981] | 19 | 0.045 | 1,025 | ··· | 11.3 | 1.41 | ··· | ≥0.24–0.47 | straight, rectangular flume; plane bed; no sidewall correction |
| | | | | *Parker and Klingeman [1982] Reference Transport Rate* | | | | | |
| *Leopold and Emmett* [1976, 1977]‖‖ | 20 | 0.072 | 32 | $\tau^*_{c_{ri}}=0.072(D_i/D_{50m})^{-0.86}$ | 1.25 | 2.8 | 2650 | 0.006 | natural, dune-ripple channel (East Fork); no sidewall or form drag correction |
| From *Wilcock and Southard* [1988] | | | | | | | | | |
| *Day* [1980] | | 0.037 | 48 | $\tau^*_{c_{ri}}=0.037(D_i/D_{50m})^{-0.81}$ | 1.82 | 2.09 | ··· | 0.03 | straight, rectangular flume; bed form type not reported; no form drag or sidewall correction (?) |
| | | 0.037 | 39 | $\tau^*_{c_{ri}}=0.037(D_i/D_{50m})^{-0.95}$ | 1.57 | 1.73 | ··· | 0.03 | as above |
| *Dhamotharan et al.* [1980] | | 0.071 | 108 | $\tau^*_{c_{ri}}=0.071(D_i/D_{50m})^{-1.1}$ | 2.16 | 1.43 | 2630 | 0.07 | straight, rectangular flume; bed form type not reported by *Wilcock and Southard* [1988]; no form drag or sidewall correction (?) |
| *Misri et al.* [1984] | | 0.048 | 101 | $\tau^*_{c_{ri}} = 0.048(D_i/D_{50m})^{-1.0}$ | 2.36 | 1.05 | 2650 | <0.05 | straight, rectangular flume; bed form type not reported; *Einstein* [1950] sidewall and form drag correction¶ |
| | | 0.042 | 194 | $\tau^*_{c_{ri}} = 0.042(D_i/D_{50m})^{-0.95}$ | 3.81 | 1.65 | 2650 | <0.07 | as above |
| | | 0.037 | 196 | $\tau^*_{c_{ri}} = 0.037(D_i/D_{50m})^{-0.92}$ | 4.00 | 1.29 | 2650 | <0.07 | as above |
| *Wilcock* [1987] | | 0.030 | 61 | $\tau^*_{c_{ri}} = 0.030(D_i/D_{50m})^{-1.0}$ | 1.83 | 0.53 | 2650 | 0.04 | straight, rectangular flume; various bed forms; *Einstein* [1950] sidewall and form drag correction¶ |
| | | 0.036 | 67 | $\tau^*_{c_{ri}} = 0.036(D_i/D_{50m})^{-0.97}$ | 1.83 | 1.06 | 2650 | 0.03 | as above |
| | | 0.023§ | 12 | $\tau^*_{c_{ri}} = 0.023(D_i/D_{50m})^{-0.98}$ | 0.67 | 0.29 | 2650 | 0.02 | as above |
| | | 0.037§ | 332 | $\tau^*_{c_{ri}} = 0.037(D_i/D_{50m})^{-1.1}$ | 5.28 | 0.20 | 2650 | 0.06 | straight, rectangular flume; plane bed; *Einstein* [1950] sidewall and form drag correction¶ |
| *Wilcock* [1992a] | 21 | 0.049 | 115 | $\tau^*_{c_{ri}} = 0.049(D_i/D_{50m})^{-1.04}$ | 2.55 | 0.89 | ··· | 0.07 | straight, rectangular flume; various bed forms; *Einstein* [1950] sidewall and form drag correction;¶ bimodal sediment |
| | | ? | ? | $\tau^*_{c_{ri}} = 0.017(D_i/D_{50m})^{-1.25}$, $0.27 \le D_i/D_{50m} \le 0.39$; $\tau^*_{c_{ri}} = 0.063(D_i/D_{50m})^{-1.14}$, $2.1 \le D_i/D_{50m} \le 3.1$ | 2.00 | 1.65 | ··· | ··· | as above, but with strongly bimodal sediment; $D_{50m}$ is fictitious, not a size occurring in the mixture |
| | | 0.052 | 19 | $\tau^*_{c_{ri}} = 0.52(D_i/D_{50m})^{-0.73}$, $0.7 \le D_i/D_{50m} \le 1.0$; $\tau^*_{c_{ri}} = 0.042(D_i/D_{50m})^{-1.17}$, $5.8 \le D_i/D_{50m} \le 8.3$ | 0.75 | 1.67 | ··· | 0.01 | as above, but $D_{50m}$ is real |
| *Kuhnle* [1993b] | | 0.035 | 8.4 | $\tau^*_{c_{ri}} = 0.035(D_i/D_{50m})^{-1.0}$ | 0.47 | 2.18 | ··· | 0.02 | straight, rectangular flume; plane bed or low amplitude bed forms; *Vanoni and Brooks* [1957] sidewall correction¶,*** |
| | | 0.039§ | 404 | $\tau^*_{c_{ri}} = 0.039(D_i/D_{50m})^{-1.1}$ | 5.58 | 0.36 | ··· | 0.12 | as above |
| | | 0.043 | 8.7 | $\tau^*_{c_{ri}} = 0.043(D_i/D_{50m})^{-1.1}$, $0.45 \le D_i/D_{50m} \le 2.5$; $\tau^*_{c_{ri}} = 0.019(D_i/D_{50m})^{-0.32}$, $3.6 \le D_i/D_{50m} \le 20.4$ | 0.47 | 0.87 | ··· | 0.02 | as above |

**Table 1a.** (Continued)

| Source | Note* | $\tau^*_{c_{r50m}}$ | $Re^*_c$† | Proposed $\tau^*_c$ Function Other Than Shields' | $D_{50m}$, mm | $\sigma_{gm}$ ($\phi$) | $\rho_s$, kg/m³ | $D_{50m}/h_c$‡ | Experimental Conditions |
|---|---|---|---|---|---|---|---|---|---|
| | | | | *Laboratory Mixture Grain Size Distribution* | | | | | |
| | | 0.041 | 11 | $\tau^*_{c_{ri}} = 0.041(D_i/D_{50m})^{-1.0}$, $0.37 \le D_i/D_{50m} \le 3.0$; $\tau^*_{c_{ri}} = 0.035(D_i/D_{50m})^{-0.55}$, $4.2 \le D_i/D_{50m} \le 11.9$ | 0.57 | 1.89 | ⋯ | 0.02 | as above |
| | | 0.045 | 29 | $\tau^*_{c_{ri}} = 0.045(D_i/D_{50m})^{-1.0}$, $0.22 \le D_i/D_{50m} \le 1.2$; $\tau^*_{c_{ri}} = 0.037(D_i/D_{50m})^{-0.42}$, $1.8 \le D_i/D_{50m} \le 7.1$ | 0.95 | 2.03 | ⋯ | 0.03 | as above |
| *Wilcock and McArdell* [1993] | 22 | 0.020 | 219 | $\tau^*_{c_{ri}} = 0.028(D_i/D_{50s})^{-0.45}$, $0.77 \le D_i/D_{50s} \le 17.3$ | 5.3 | 2.88 | 2610 | 0.16 | straight, rectangular flume; variable bed forms; *Einstein* [1950] sidewall and form drag correction¶ |
| | | | | *Other Reference Transport Definition* | | | | | |
| From *Bridge and Dominic* [1984] | | | | | | | | | |
| *Gilbert* [1914]†† | | 0.040§ | 4.2 | ⋯ | 0.30 | <0.09 | 2690 | 0.08 | straight, rectangular flume; upper stage plane bed; *Williams* [1970] sidewall correction; subangular river sand |
| | | 0.052§ | 6.9 | ⋯ | 0.38 | <0.11 | 2690 | 0.13 | as above |
| | | 0.042§ | 9.6 | ⋯ | 0.51 | <0.26 | 2690 | 0.17 | as above |
| | | 0.030 | 16 | ⋯ | 0.79 | <0.32 | 2690 | 0.24 | as above |
| | | 0.040 | 145 | ⋯ | 3.17 | <0.24 | 2690 | 0.24 | as above, but lower stage plane bed and subrounded river gravel |
| | | 0.041§ | 286 | ⋯ | 4.94 | <0.26 | 2690 | 0.19 | as above |
| *Williams* [1970] | | 0.040 | 36 | ⋯ | 1.35 | 0.20 | ⋯ | 0.40 | straight, rectangular flume; upper stage plane bed; *Williams* [1970] sidewall correction |
| | | 0.040§ | 41 | ⋯ | 1.35 | 0.20 | ⋯ | 0.07 | as above, but lower stage plane bed |
| *Guy et al*. [1966] | | 0.040§ | 2.1 | ⋯ | 0.19 | 0.45 | ⋯ | 0.02 | straight, rectangular flume; upper stage plane bed; no sidewall correction |
| | | 0.040 | 3.3 | ⋯ | 0.28 | 0.81 | ⋯ | 0.03 | as above |
| | | 0.040 | 4.4 | ⋯ | 0.32 | 0.65 | ⋯ | 0.07 | as above |
| | | 0.040 | 22 | ⋯ | 0.93 | 0.59 | ⋯ | 0.008 | as above, but lower stage plane bed |

| Source | Note* | $\tau^*_{c_{r50ss}}$ | $Re^*_c$† | Proposed $\tau^*_c$ Function Other Than Shields' | $D_{50ss}$, mm | $\sigma_{gss}$ ($\phi$) | $\rho_s$, kg/m³ | $D_{50ss}/h_c$‡ | Experimental Conditions |
|---|---|---|---|---|---|---|---|---|---|
| | | | | *Subsurface Grain Size Distribution* | | | | | |
| | | | | *Parker and Klingeman [1982] Reference Transport Rate* | | | | | |
| *Parker and Klingeman* [1982] | | 0.088 | 2,410 | $\tau^*_{c_{ri}} = 0.088(D_i/D_{50ss})^{-0.98}$ | 20 | 2.20 | 2850 | 0.06 | same as *Parker and Klingeman* [1982], first entry of Table 1a |
| From *Wilcock and Southard* [1988] | | | | | | | | | |
| *Milhous* [1973] | | 0.073 | 2,113 | $\tau^*_{c_{ri}} = 0.073(D_i/D_{50ss})^{-0.98}$ | 19.5 | 2.20 | 2850 | 0.07 | as above |
| *Kuhnle* [1992] | | 0.086 | 596 | $\tau^*_{c_{ri}} = 0.086(D_i/D_{50ss})^{-0.81}$ | 8.31 | 2.77 | ⋯ | 0.02 | same as *Kuhnle* [1992], seventh entry of Table 1a |
| *Kuhnle* [1993b] | | 0.089 | 606 | $\tau^*_{c_{ri}} = 0.080(D_i/D_{50ss})^{-0.94}$, $0.04 \le D_i/D_{50m} \le 0.34$; $\tau^*_{c_{ri}} = 0.089(D_i/D_{50ss})^{-0.81}$, $0.67 \le D_i/D_{50m} \le 5.4$ | 8.31 | 2.77 | ⋯ | 0.02 | as above seventh entry of Table 1a |

Note that symbols for similar footnotes may be different in Tables 1a–1e. While most $\tau^*_{c_{r50}}$ values are determined from extrapolation of bed load transport rates, some are based on extrapolation of particle or bed form velocity [e.g., *Ippen and Verma*, 1953; *Meland and Norrman*, 1966; *Sternberg*, 1971]. See notation section for symbols not previously defined in text. See respective appendix notes for values in parentheses. Here "u" denotes uniform grain sizes ($\sigma_g \leq 0.5$), and "m" denotes mixed grain sizes ($\sigma_g > 0.5$), where $\sigma_g$ is the graphic standard deviation defined as $(\phi_{84} - \phi_{16})/2$ [*Folk*, 1974].

*See appendix.

†$Re^*_c = u^*_c D_{50}/\nu$. Most values are back-calculated from reported data. For example, using $\tau^*_{c_{r50s}}$ and $D_{50s}$ reported by *Parker and Klingeman* [1982], we calculated $\tau_{c_{r50s}}$ from *Shields*' [1936] equation, allowing determination of $u^*_c = (\tau_{c_{r50s}}/\rho)^{0.5}$, and hence $Re^*_c$, with $\nu$ estimated from water temperatures reported by *Milhous* [1973]. Where unreported by the original source, it was assumed that $\rho_s$ and $\rho$ were 2650 and 1000 kg/m$^3$, respectively, and that $\nu = 10^{-6}$ m$^2$/s for laboratory and theoretical studies and $1.5 \times 10^{-6}$ m$^2$/s for field studies.

‡Where unreported by a source, $h_c$ values are back-calculated from the *Shields* [1936] equation with the $\tau_c$ expressed as a depth-slope product ($h_c = \tau^*_c (\rho_s - \rho) D_{50}/\rho_S$, with $S$ determined from the original source, where available). For example, we used the average slope of *Gilbert*'s [1914] 7.01-mm experiments for the first Gilbert entry from *Shields* [1936]; for Gilbert entries from *Bridge and Dominic* [1984] we used the subset of slopes corresponding with plane bed morphologies for each sediment. This procedure may cause overestimation of $D_{50}/h_c$ where depth-slope products have been reduced for roughness effects.

§Used in Plate 1.

‖Reported data are with respect to the geometric mean grain size.

¶Use of the average velocity in the *Einstein* [1942; 1950], *Johnson* [1942] and *Vanoni and Brooks* [1957] equations likely overestimates $\tau'$ (see note 3).

**Here we describe grain protrusion in terms of projection and exposure [sensu *Kirchner et al.*, 1990].

††Reported data are with respect to the mean nominal grain diameter. Nominal diameters are assumed equivalent to intermediate grain diameters [*Cui and Komar*, 1984]. Mean and median sizes are similar for near-uniform sediment.

‡‡Sidewall correction for the proximity of walls (i.e., $W/h$), but not for the difference in wall and bed grain roughness.

§§Reported data are with respect to mean grain sizes. Mean and median grain sizes are assumed similar for near-uniform sediment.

‖‖Reported data are determined from bulk (i.e., surface and subsurface mixture) grain size sampling, treated here as mixture-based values.

¶¶Also given by *Ashida and Bayazit* [1973].

***Sidewall correction for the difference in wall and bed grain roughness but not for the proximity of walls (i.e., $W/h$).

**Table 1b.** Previously Reported $\tau^*_{c_{50}}$ Values: Visual Observation

| | | | | | Surface Grain Size Distribution | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Source | Note* | $\tau^*_{c_{v50s}}$ | $Re^*_c$† | Proposed $\tau^*_c$ Function Other Than Shields' | $D_{50s}$, mm | $\sigma_{gs}$ ($\phi$) | $\rho_s$, kg/m$^3$ | $D_{50s}/h_c$‡ | Experimental Conditions |
| | | | | *Various Movement Definitions* | | | | | |
| *Coleman* [1967] | 23 | 0.284 (0.005) | 6.2 | ··· | 12.7 | 0 | 1278 | ··· | straight rectangular flume; sidewall correction implicit; plastic test grain on fixed plane bed of like grains; significant projection and exposure;§ saddle rotation; shear measured with strain gauge; water or sodium carboxymethyl-cellulose fluid medium |
| | | 0.284 (0.005) | 7.3 | ··· | 12.7 | 0 | 1278 | ··· | as above |
| | | 0.255 (0.004) | 7.3 | ··· | 12.7 | 0 | 1278 | ··· | as above |
| | | 0.230 (0.004) | 10 | ··· | 12.7 | 0 | 1278 | ··· | as above |
| | | 0.195 (0.004) | 13 | ··· | 12.7 | 0 | 1278 | ··· | as above |
| | | 0.166 (0.004) | 12 | ··· | 12.7 | 0 | 1278 | ··· | as above |
| | | 0.160 (0.004) | 15 | ··· | 12.7 | 0 | 1278 | ··· | as above |
| | | 0.142 (0.004) | 19 | ··· | 12.7 | 0 | 1278 | ··· | as above |
| | | 0.121 (0.005) | 31 | ··· | 12.7 | 0 | 1278 | ··· | as above |
| | | 0.113 (0.009) | 130 | ··· | 12.7 | 0 | 1278 | ··· | as above |
| | | 0.119 (0.011) | 160 | ··· | 12.7 | 0 | 1278 | ··· | as above |
| | | 0.129 (0.011) | 190 | ··· | 12.7 | 0 | 1278 | ··· | as above |
| | | 0.138 (0.014) | 380 | ··· | 12.7 | 0 | 1278 | ··· | as above |

**Table 1b.** (continued)

| Source | Note* | $\tau^*_{c_{v50s}}$ | $Re^*_c$† | Proposed $\tau^*_c$ Function Other Than Shields' | Surface Grain Size Distribution: $D_{50s}$, mm | $\sigma_{gs}$ ($\phi$) | $\rho_s$, kg/m$^3$ | $D_{50s}/h_c$‡ | Experimental Conditions |
|---|---|---|---|---|---|---|---|---|---|
| | | 0.123(0.012) | 360 | ⋯ | 12.7 | 0 | 1278 | ⋯ | as above |
| | | 0.116(0.004) | 19 | ⋯ | 12.7 | 0 | 7822 | ⋯ | as above, but with steel test grain |
| | | 0.117(0.004) | 23 | ⋯ | 12.7 | 0 | 7822 | ⋯ | as above |
| | | 0.084(0.004) | 43 | ⋯ | 12.7 | 0 | 7822 | ⋯ | as above |
| | | 0.098(0.006) | 58 | ⋯ | 12.7 | 0 | 7822 | ⋯ | as above |
| | | 0.099(0.007) | 82 | ⋯ | 12.7 | 0 | 7822 | ⋯ | as above |
| | | 0.108(0.010) | 140 | ⋯ | 12.7 | 0 | 7822 | ⋯ | as above |
| | | 0.107(0.011) | 500 | ⋯ | 12.7 | 0 | 7822 | ⋯ | as above |
| | | 0.116(0.012) | 610 | ⋯ | 12.7 | 0 | 7822 | ⋯ | as above |
| | | 0.107(0.011) | 1340 | ⋯ | 12.7 | 0 | 7822 | ⋯ | as above |
| *Fenton and Abbott* [1977] | | 0.248 | 66 | ⋯ | 2.5 | 0? | ⋯ | ⋯ | straight rectangular flume; angular polystyrene grain on fixed plane bed of like grains;** sidewall correction implicit; protrusion of −0.20 |
| | | 0.164 | 54 | ⋯ | 2.5 | 0? | ⋯ | ⋯ | as above, but with protrusion of −0.06 |
| | | 0.085 | 39 | ⋯ | 2.5 | 0? | ⋯ | ⋯ | as above, but with protrusion of 0.30 |
| | | 0.050 | 30 | ⋯ | 2.5 | 0? | ⋯ | ⋯ | as above |
| | | 0.040 | 27 | ⋯ | 2.5 | 0? | ⋯ | ⋯ | as above, but with protrusion of 0.34 |
| | | 0.149 | 51 | ⋯ | 2.5 | 0? | ⋯ | ⋯ | as above, but with protrusion of 0.04 |
| | | 0.119 | 46 | ⋯ | 2.5 | 0? | ⋯ | ⋯ | as above, but with protrusion of 0.16 |
| | | 0.070 | 35 | ⋯ | 2.5 | 0? | ⋯ | ⋯ | as above, but with protrusion of 0.25 |
| | | 0.072 | 72 | ⋯ | 2.5 | 0? | ⋯ | ⋯ | as above, but with lead-filled test grains and protrusion of 0.34 |
| | | 0.032 | 48 | ⋯ | 2.5 | 0? | ⋯ | ⋯ | as above, but with protrusion of 0.41 |
| | | 0.083 | 78 | ⋯ | 2.5 | 0? | ⋯ | ⋯ | as above, but with protrusion of 0.08 |
| | | 0.107 | 88 | ⋯ | 2.5 | 0? | ⋯ | ⋯ | as above, but with protrusion of 0.05 |
| | | 0.053 | 61 | ⋯ | 2.5 | 0? | ⋯ | ⋯ | as above, but with protrusion of 0.44 |
| | | 0.071 | 70 | ⋯ | 2.5 | 0? | ⋯ | ⋯ | as above, but with protrusion of 0.35 |
| | | 0.076 | 73 | ⋯ | 2.5 | 0? | ⋯ | ⋯ | as above, but with protrusion of 0.16 |
| | | 0.009 | 1690 | ⋯ | 38 | 0 | ⋯ | 0.23 | straight rectangular flume; fixed plane bed of wooden grains; table tennis test grains filled with lead shot or polystyrene grains and sand; sidewall correction implicit; protrusion of 0.82 |
| | | 0.010 | 1700 | ⋯ | 38 | 0 | ⋯ | 0.23 | as above |
| | | 0.010 | 1760 | ⋯ | 38 | 0 | ⋯ | 0.36 | as above |
| | | 0.011 | 3200 | ⋯ | 38 | 0 | ⋯ | 0.36 | as above |
| | | 0.012 | 3280 | ⋯ | 38 | 0 | ⋯ | 0.24 | as above |
| *Petit* [1994] | | 0.058 | 1403 | $\tau^*_{c_{vi}} = 0.058(D_i/D_{50s})^{-0.66}$ | 12.8 | <0.20? | 2650 | ⋯ | straight rectangular flume; natural grains on plane bed of like grains; *Vanoni and Brooks* [1957] sidewall correction‖,¶ |
| | | 0.047 | 3284 | $\tau^*_{c_{vi}} = 0.047(D_i/D_{50s})^{-0.73}$ | 24.2 | <0.34? | 2650 | ⋯ | as above |
| | | 0.045 | 6624 | $\tau^*_{c_{vi}} = 0.045(D_i/D_{50s})^{-0.81}$ | 39.2 | <0.15? | 2650 | ⋯ | as above |
| | | 0.049 | 2444 | $\tau^*_{c_{vi}} = 0.049(D_i/D_{50s})^{-0.68}$ | 19.6 | <0.54? | 2650 | ⋯ | as above, but with fixed bed |

| Source | Note* | $\tau^*_{c_{v50m}}$ | $Re^*_c$† | Proposed $\tau^*_c$ Function Other Than Shields' | Laboratory Mixture Grain Size Distribution: $D_{50m}$, mm | $\sigma_{gm}$ ($\phi$) | $\rho_s$, kg/m$^3$ | $D_{50m}/h_c$‡ | Experimental Conditions |
|---|---|---|---|---|---|---|---|---|---|
| | | | | *"Medium Movement" [Kramer, 1935] or Its Equivalent* | | | | | |
| *Gilbert* [1914]** | 24 | 0.052†† | 322 | ··· | 4.94 | <0.26 | 2690 | 0.11 | straight, rectangular flume; plane bed; *Shimizu* [1989] sidewall correction;‡‡ Subrounded grains |
| | | 0.052†† | 323 | ··· | 4.94 | <0.26 | 2690 | 0.16 | as above |
| | | 0.069†† | 372 | ··· | 4.94 | <0.26 | 2690 | 0.15 | as above |
| | | 0.048†† | 310 | ··· | 4.94 | <0.26 | 2690 | 0.17 | as above |
| | | 0.058†† | 341 | ··· | 4.94 | <0.26 | 2690 | 0.18 | as above |
| | | 0.046†† | 304 | ··· | 4.94 | <0.26 | 2690 | 0.12 | as above |
| | | 0.047†† | 308 | ··· | 4.94 | <0.26 | 2690 | 0.16 | as above |
| | | 0.062†† | 352 | ··· | 4.94 | <0.26 | 2690 | 0.18 | as above |
| | | 0.053 | 326 | ··· | 4.94 | <0.26 | 2690 | 0.27 | as above |
| | | 0.032†† | 52 | ··· | 1.71 | <0.48 | 2690 | 0.09 | as above |
| | | 0.057†† | 69 | ··· | 1.71 | <0.48 | 2690 | 0.20 | as above |
| | | 0.049†† | 160 | ··· | 3.17 | <0.24 | 2690 | 0.11 | as above |
| | | 0.049†† | 161 | ··· | 3.17 | <0.24 | 2690 | 0.11 | as above |
| | | 0.060†† | 587 | ··· | 7.01 | <0.22 | 2690 | 0.08 | as above |
| *Kramer* [1935] | | 0.043 | 8.2 | ··· | 0.53 | 0.81 | 2700 | 0.03 | straight, rectangular flume; plane bed; *Shimizu* [1989] sidewall correction;‡‡ well-rounded grains |
| | | 0.048 | 8.7 | ··· | 0.53 | 0.81 | 2700 | 0.02 | as above |
| | | 0.039 | 7.7 | ··· | 0.53 | 0.81 | 2700 | 0.02 | as above |
| | | 0.040 | 8.0 | ··· | 0.53 | 0.81 | 2700 | 0.01 | as above |
| | | 0.037 | 7.3 | ··· | 0.51 | 0.74 | 2700 | 0.04 | as above |
| | | 0.037 | 7.1 | ··· | 0.51 | 0.74 | 2700 | 0.02 | as above |
| | | 0.034 | 7.0 | ··· | 0.51 | 0.74 | 2700 | 0.02 | as above |
| | | 0.030 | 6.5 | ··· | 0.51 | 0.74 | 2700 | 0.02 | as above |
| | | 0.035 | 7.8 | ··· | 0.55 | 0.62 | 2700 | 0.04 | as above |
| | | 0.039 | 8.2 | ··· | 0.55 | 0.62 | 2700 | 0.02 | as above |
| | | 0.038 | 7.9 | ··· | 0.55 | 0.62 | 2700 | 0.02 | as above |
| | | 0.037 | 8.0 | ··· | 0.55 | 0.62 | 2700 | 0.01 | as above |
| *USWES* [1935] | | 0.052 | 9.5 | ··· | 0.43 | 0.95 | 2650 | 0.01 | straight, rectangular flume; plane bed; *Shimizu* [1989] sidewall correction;‡‡ subangular to subrounded river sand |
| | | 0.044 | 8.8 | ··· | 0.43 | 0.95 | 2650 | 0.02 | as above |
| | | 0.052 | 9.6 | ··· | 0.43 | 0.95 | 2650 | 0.02 | as above |
| | | 0.051 | 10 | ··· | 0.45 | 0.66 | 2650 | 0.01 | as above |
| | | 0.067 | 12 | ··· | 0.45 | 0.66 | 2650 | 0.01 | as above |
| | | 0.043 | 8.0 | ··· | 0.48 | 0.53 | 2650 | 0.01 | as above, but subrounded to rounded grains |
| | | 0.048 | 9.0 | ··· | 0.48 | 0.53 | 2650 | 0.02 | as above |
| | | 0.049 | 8.6 | ··· | 0.48 | 0.53 | 2650 | 0.02 | as above |
| | | 0.052 | 7.8 | ··· | 0.44 | 0.82 | 2650 | 0.01 | same as above, but angular to subrounded grains |
| | | 0.049 | 7.6 | ··· | 0.44 | 0.82 | 2650 | 0.02 | as above |
| | | 0.063 | 8.2 | ··· | 0.44 | 0.82 | 2650 | 0.02 | as above |
| | | 0.053 | 6.6 | ··· | 0.40 | 0.69 | 2650 | 0.01 | as above, but angular to subangular grains |
| | | 0.053 | 6.7 | ··· | 0.40 | 0.69 | 2650 | 0.02 | as above |
| | | 0.059 | 7.0 | ··· | 0.40 | 0.69 | 2650 | 0.02 | as above |
| | | 0.048†† | 5.5 | ··· | 0.34 | 0.37 | 2650 | 0.02 | as above, but subrounded to subangular grains |

**Table 1b.** (Continued)

| Source | Note* | $\tau^*_{c_{v50m}}$ | $Re^*_c$† | Proposed $\tau^*_c$ Function Other Than Shields' | $D_{50m}$, mm | $\sigma_{gm}$ ($\phi$) | $\rho_s$, kg/m$^3$ | $D_{50m}/h_c$‡ | Experimental Conditions grains |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | Laboratory Mixture Grain Size Distribution | | | | |
| | | 0.076 | 5.1 | ··· | 0.25 | 0.53 | 2650 | 0.008 | as above |
| | | 0.038 | 196 | ··· | 4.0 | 0.56 | 2650 | 0.04 | as above |
| | | 0.042 | 200 | ··· | 4.0 | 0.56 | 2650 | 0.05 | as above |
| | | 0.045 | 212 | ··· | 4.0 | 0.56 | 2650 | 0.06 | as above |
| | | 0.074†† | 2.4 | ··· | 0.18 | 0.32 | 2650 | 0.008 | as above, but subangular to angular grains |
| *Chang* [1939]** | | 0.061 | 1.5 | ··· | 0.134 | | 2520 | 0.0008 | straight, rectangular flume, and convergent-walled flume; plane bed; *Shimizu* [1989] sidewall correction‡‡ |
| | | | | *"General Movement" [Kramer, 1935]* | | | | | |
| From *Kramer* [1935] | | | | | | | | | |
| *Schaffernak* [1916] | | 0.030 | 41 | ··· | 1.5 | 0.5 | 2650 | ··· | unreported by *Kramer* [1935], but experimental procedure directly comparable to his [*Kramer*, 1935] |
| H. Krey (unpublished Elbe experiments report) | | 0.057 | 14 | ··· | 0.60 | 0.94 | 2650 | ··· | as above |
| | | 0.060 | 13 | ··· | 0.57 | 0.69 | 2650 | ··· | as above |
| | | 0.056 | 15 | ··· | 0.63 | 0.89 | 2650 | ··· | as above |
| | | 0.053 | 13 | ··· | 0.57 | 0.71 | 2650 | ··· | as above |
| | | 0.034 | 21 | ··· | 0.92 | 1.20 | 2650 | ··· | as above, but less comparable [*Kramer*, 1935] |
| | | 0.038 | 14 | ··· | 0.67 | 1.19 | 2650 | ··· | as above |
| | | 0.069 | 3.2 | ··· | 0.21 | 0.67 | 2650 | ··· | as above |
| H. Krey (unpublished report) | | 0.038 | 5.8 | ··· | 0.38 | 0.21 | 2680 | ··· | as above |
| | | 0.033 | 8.8 | ··· | 0.53 | 0.18 | 2610 | ··· | as above |
| | | 0.025 | 14 | ··· | 0.8 | 0.24 | 2570 | ··· | as above |
| *Schoklitsch* [1914] | | 0.052 | 476 | ··· | 6.52 | 0 | 2600 | ··· | straight, rectangular flume [*Mantz*, 1977]; slate grains (?); experimental procedure less comparable [*Kramer*, 1935] |
| | | 0.041 | 206 | ··· | 4.05 | 0 | 2600 | ··· | as above |
| | | 0.031 | 75 | ··· | 2.26 | 0 | 2600 | ··· | as above |
| | | 0.022 | 26 | ··· | 1.24 | 0 | 2600 | ··· | as above |
| | | 0.019 | 15 | ··· | 0.92 | 0 | 2600 | ··· | as above |
| *Engels* [1932] | | 0.061 | 31 | ··· | 1.00 | 1.35 | 2650 | <0.02 | meandering, large-scale flume [*Engels and Kramer*, 1932]; plane bed; no correction for sidewalls or channel curvature (?); experimental procedure less comparable [*Kramer*, 1935] |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| *Other Visual Movement Definition* | | | | | | | | | |
| From *O'Brien and Rindlaub* [1934] and *Mavis et al.* [1935] | | | | | | | | | |
| *Ho* [1933] | | 0.015 | 5 | ··· | 0.5 | ≤0.25 | 2620 | ··· | straight, rectangular flume; plane bed; no sidewall correction (?); river sand |
| | | 0.014 | 9 | ··· | 0.7 | ≤0.25 | 2680 | ··· | as above |
| | | 0.024 | 20 | ··· | 1.0 | ≤0.25 | 2630 | ··· | as above |
| | | 0.028 | 35 | ··· | 1.4 | ≤0.25 | 2630 | ··· | as above |
| | | 0.030 | 62 | ··· | 2.0 | ≤0.25 | 2620 | ··· | as above |
| | | 0.029 | 102 | ··· | 2.8 | ≤0.25 | 2660 | ··· | as above |
| | | 0.024 | 159 | ··· | 4.0 | ≤0.25 | 2650/2660 | ··· | as above, but uncertain whether grains are river sand or crushed limestone |
| | | 0.036 | 194 | ··· | 4.0 | ≤0.25 | 2650/2660 | ··· | as above |
| | | 0.028 | 287 | ··· | 5.7? | ≤0.25 | 2650/2660 | ··· | as above |
| | | 0.028 | 288 | ··· | 5.7 | ≤0.25 | 2650/2660 | ··· | as above |
| From *Mavis et al.* [1937] | | | | | | | | | |
| *Liu* [1935] | | 0.022†† | 198 | ··· | 4.3 | 0.31 | 2660 | 0.03 | straight, rectangular flume; plane bed; *Shimizu* [1989] sidewall correction‡‡ |
| | | 0.026†† | 213 | ··· | 4.3 | 0.31 | 2660 | 0.05 | as above |
| | | 0.024†† | 207 | ··· | 4.3 | 0.31 | 2660 | 0.12 | as above |
| | | 0.032†† | 235 | ··· | 4.3 | 0.31 | 2660 | 0.19 | as above |
| | | 0.019†† | 110 | ··· | 3.1 | 0.33 | 2660 | 0.04 | as above |
| | | 0.028†† | 135 | ··· | 3.1 | 0.33 | 2660 | 0.05 | as above |
| | | 0.029†† | 137 | ··· | 3.1 | 0.33 | 2660 | 0.10 | as above |
| | | 0.037†† | 156 | ··· | 3.1 | 0.33 | 2660 | 0.16 | as above |
| | | 0.024†† | 75 | ··· | 2.2 | 0.35 | 2660 | 0.03 | as above |
| | | 0.028†† | 82 | ··· | 2.2 | 0.35 | 2660 | 0.05 | as above |
| | | 0.029†† | 82 | ··· | 2.2 | 0.35 | 2660 | 0.10 | as above |
| | | 0.035†† | 90 | ··· | 2.2 | 0.35 | 2660 | 0.17 | as above |
| | | 0.022†† | 37 | ··· | 1.4 | 0.31 | 2660 | 0.03 | as above |
| | | 0.023†† | 37 | ··· | 1.4 | 0.31 | 2660 | 0.06 | as above |
| | | 0.026†† | 39 | ··· | 1.4 | 0.31 | 2660 | 0.11 | as above |
| | | 0.030†† | 43 | ··· | 1.4 | 0.31 | 2660 | 0.20 | as above |
| | | 0.021†† | 158 | ··· | 3.8 | 0.36 | 2660 | 0.03 | as above |
| | | 0.023†† | 165 | ··· | 3.8 | 0.36 | 2660 | 0.06 | as above |
| | | 0.033†† | 198 | ··· | 3.8 | 0.36 | 2660 | 0.09 | as above |
| | | 0.034†† | 204 | ··· | 3.8 | 0.36 | 2660 | 0.17 | as above |
| | | 0.026†† | 53 | ··· | 1.7 | 0.45 | 2660 | 0.03 | as above |
| | | 0.028†† | 55 | ··· | 1.7 | 0.45 | 2660 | 0.05 | as above |
| | | 0.026†† | 53 | ··· | 1.7 | 0.45 | 2660 | 0.11 | as above |
| | | 0.030‡‡ | 57 | ··· | 1.7 | 0.45 | 2660 | 0.20 | as above |
| *White* [1940] | 25 | 0.338†† | 0.043 | ··· | 0.21 | ~u? | ··· | 0.05 | straight, rectangular flume; plane bed; no sidewall correction; oil fluid medium; entirely laminar flow |
| | | 0.168†† | 0.30 | ··· | 0.90 | ~u? | ··· | 0.11 | as above |
| | | 0.180 | 2.4 | ··· | 0.122 | ~u | ··· | ··· | convergent-walled nozzle; plane bed; no sidewall correction; water fluid medium; laminar boundary layer within steady "inviscid" flow |
| | | 0.119 | 33 | ··· | 0.90 | ~u | 2600 | ··· | as above, but with turbulent boundary layer |
| | | 0.101 | 480 | ··· | 5.6 | ~u | 2600 | ··· | as above |
| | | 0.064 | 35 | ··· | 0.71 | m | 7900 | ··· | as above, but with steel shot grains |

**Table 1b.** (Continued)

| Source | Note* | $\tau^*_{c_{i50m}}$ | $Re^*_c$† | Proposed $\tau^*_c$ Function Other Than Shields' | $D_{50m}$, mm | $\sigma_{gm}$ ($\phi$) | $\rho_s$, kg/m$^3$ | $D_{50m}/h_c$‡ | Experimental Conditions |
|---|---|---|---|---|---|---|---|---|---|
| | | 0.098 | 80 | ··· | 0.90 | ~u | 2100 | ··· | as above, but with natural grains and fluid medium of air |
| | | 0.102 | 1280 | ··· | 5.6 | ~u | 2100 | ··· | as above |
| *Meyer-Peter and Müller* [1948]‖‖ | | 0.033†† | 133 | $\tau^*_{c_{i50m}} = [\tau_{c_{i50m}}(Q_b/Q)(n_g/n_b)^{3/2}]/[(\rho_s - \rho)gD_{50m}]$ | 3.2 | <0.12? | 2680 | 0.01 | straight, rectangular flume; plane bed or very low amplitude bed forms; form drag and sidewall correction; rounded grains |
| | | 0.032†† | 101 | as above | 2.7 | <0.15? | 2680 | 0.02 | as above |
| | | 0.037†† | 183 | as above | 3.8 | <0.16? | 2680 | 0.02 | as above |
| | | 0.030†† | 151 | as above | 3.6 | <0.15? | 2680 | 0.01 | as above |
| | | 0.039†† | 178 | as above | 3.66 | <0.13? | 2680 | 0.03 | as above |
| | | 0.040†† | 179 | as above | 3.66 | <0.13? | 2680 | 0.06 | as above |
| | | 0.037†† | 609 | as above | 8.5 | <0.19? | 2680 | 0.06 | as above |
| | | 0.050†† | 575 | as above | 7.4 | <0.28? | 2680 | 0.10 | as above |
| | | 0.047†† | 686 | as above | 8.5 | <0.23? | 2680 | 0.10 | as above |
| | | 0.040†† | 142 | as above | 3.14 | <0.15? | 2680 | 0.13 | as above |
| | | 0.033†† | 59 | as above | 1.86 | <0.33? | 2680 | 0.01 | as above, but with angular grains |
| | | 0.025†† | 113 | as above | 3.14 | <0.15? | 2680 | 0.01 | as above |
| | | 0.033†† | 127 | as above | 3.1 | <0.17? | 2680 | 0.02 | as above |
| | | 0.038†† | 176 | as above | 3.66 | <0.13? | 2680 | 0.03 | as above |
| | | 0.040†† | 137 | as above | 3.06 | <0.19? | 2680 | 0.02 | as above |
| | | 0.040†† | 106 | as above | 2.58 | <0.18? | 2680 | 0.02 | as above |
| | | 0.048†† | 119 | as above | 2.61 | <0.20? | 2680 | 0.05 | as above |
| | | 0.050†† | 233 | as above | 4.04 | <0.22? | 2680 | 0.05 | as above |
| | | 0.043†† | 241 | as above | 4.34 | <0.22? | 2680 | 0.10 | as above |
| | | 0.043†† | 81 | as above | 2.10 | <0.16? | 2680 | 0.11 | as above |
| | | 0.043†† | 147 | as above | 3.14 | <0.15? | 2680 | 0.10 | as above |
| *Wolman and Brush* [1961] | | 0.020†† | 9 | ··· | 0.67 | 0.26 | ··· | 0.05 | straight, rectangular flume; plane bed; erodible banks; *Shimizu* [1989] sidewall correction‡‡ |
| | | 0.024†† | 10 | ··· | 0.67 | 0.26 | ··· | 0.07 | as above |
| | | 0.030†† | 12 | ··· | 0.67 | 0.26 | ··· | 0.07 | as above |
| | | 0.024†† | 11 | ··· | 0.67 | 0.26 | ··· | 0.12 | as above |
| | | 0.048†† | 86 | ··· | 2 | 0.5 | ··· | 0.16 | as above |
| | | 0.049†† | 91 | ··· | 2 | 0.5 | ··· | 0.06 | as above |
| | | 0.052†† | 94 | ··· | 2 | 0.5 | ··· | 0.12 | as above |
| | | 0.036†† | 80 | ··· | 2 | 0.5 | ··· | 0.05 | as above |
| | | 0.044†† | 86 | ··· | 2 | 0.5 | ··· | 0.09 | as above |
| | | 0.030†† | 71 | ··· | 2 | 0.5 | ··· | 0.04 | as above |
| *Raudkivi* [1963]§§ | | 0.036†† | 6.0 | ··· | 0.40 | 0.42 | 2600 | 0.003 | straight, rectangular flume; plane bed; *Vanoni and Brooks* [1957] sidewall correction‖,¶ |
| *Vanoni* [1964]§§ | | 0.097†† | 1.3 | ··· | 0.102 | 0.19 | 2650 | 0.0009 | straight rectangular flume; plane bed; sidewall correction implicit; quartz grains |
| | | 0.126†† | 1.5 | ··· | 0.102 | 0.19 | 2650 | 0.001 | as above |
| | | 0.120†† | 1.5 | ··· | 0.102 | 0.19 | 2650 | 0.001 | as above |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 0.226†† | 0.43 | ⋯ | 0.037 | 0.25 | 2490 | 0.0005 | as above, but with glass beads |
| | 0.228†† | 0.43 | ⋯ | 0.037 | 0.25 | 2490 | 0.0005 | as above |
| *Neill* [1967]** | 0.030†† | 328 | ⋯ | 6.2 | ~0.48? | 2540 | 0.04 | straight, rectangular flume; plane bed; *Shimizu* [1989] sidewall correction‡‡ |
| | 0.032†† | 340 | ⋯ | 6.2 | ~0.48? | 2540 | 0.04 | as above |
| | 0.030†† | 327 | ⋯ | 6.2 | ~0.48? | 2540 | 0.05 | as above |
| | 0.041†† | 382 | ⋯ | 6.2 | ~0.48? | 2540 | 0.05 | as above |
| | 0.038†† | 372 | ⋯ | 6.2 | ~0.48? | 2540 | 0.06 | as above |
| | 0.032†† | 339 | ⋯ | 6.2 | ~0.48? | 2540 | 0.07 | as above |
| | 0.036†† | 359 | ⋯ | 6.2 | ~0.48? | 2540 | 0.07 | as above |
| | 0.032†† | 337 | ⋯ | 6.2 | ~0.48? | 2540 | 0.11 | as above |
| | 0.023†† | 289 | ⋯ | 6.2 | ~0.48? | 2540 | 0.12 | as above |
| | 0.050†† | 675 | ⋯ | 8.5 | ~0.48? | 2520 | 0.05 | as above |
| | 0.056†† | 713 | ⋯ | 8.5 | ~0.48? | 2520 | 0.07 | as above |
| | 0.037†† | 584 | ⋯ | 8.5 | ~0.48? | 2520 | 0.14 | as above |
| | 0.039 | 595 | ⋯ | 8.5 | ~0.48? | 2520 | 0.28 | as above |
| | 0.037†† | 2099 | ⋯ | 20.0 | ~0.48? | 2520 | 0.11 | as above |
| | 0.031†† | 1938 | ⋯ | 20.0 | ~0.48? | 2520 | 0.14 | as above |
| | 0.036†† | 2066 | ⋯ | 20.0 | ~0.48? | 2520 | 0.18 | as above |
| | 0.031 | 1935 | ⋯ | 20.0 | ~0.48? | 2520 | 0.23 | as above |
| | 0.027 | 1802 | ⋯ | 20.0 | ~0.48? | 2520 | 0.33 | as above |
| | 0.032 | 1950 | ⋯ | 20.0 | ~0.48? | 2520 | 0.39 | as above |
| | 0.031 | 1928 | ⋯ | 20.0 | ~0.48? | 2520 | 0.55 | as above |
| | 0.037†† | 259 | ⋯ | 5.0 | 0 | 2490 | 0.03 | as above, but with glass grains |
| | 0.039†† | 266 | ⋯ | 5.0 | 0 | 2490 | 0.04 | as above |
| | 0.042†† | 278 | ⋯ | 5.0 | 0 | 2490 | 0.04 | as above |
| | 0.038†† | 264 | ⋯ | 5.0 | 0 | 2490 | 0.04 | as above |
| | 0.037†† | 261 | ⋯ | 5.0 | 0 | 2490 | 0.05 | as above |
| | 0.027†† | 221 | ⋯ | 5.0 | 0 | 2490 | 0.08 | as above |
| *Rathburn and Guy* [1967] | 0.026 | 3.4 | ⋯ | 0.3 | 1.26 | 2650 | 0.005 | straight, rectangular flume; plane bed *Vanoni and Brooks* [1957] sidewall correction ‖,¶ |
| *Ward* [1968]‖‖ | 0.102 | 21 | ⋯ | 0.70 | <0.5 | 2640 | ⋯ | convergent-walled flume; plane bed; sidewall correction implicit; water fluid medium |
| | 0.084 | 56 | ⋯ | 1.43 | <0.5 | 2640 | ⋯ | as above |
| | 0.092 | 119 | ⋯ | 2.29 | <0.5 | 2640 | ⋯ | as above |
| | 0.110 | 25 | ⋯ | 0.70 | <0.5 | 3100 | ⋯ | as above, but with taconite grains |
| | 0.075 | 85 | ⋯ | 1.80 | <0.5 | 3100 | ⋯ | as above |
| | 0.079 | 125 | ⋯ | 2.29 | <0.5 | 3100 | ⋯ | as above |
| | 0.098 | 21 | ⋯ | 0.70 | <0.5 | 2710 | ⋯ | as above but with Douglas sand #2 |
| | 0.086 | 56 | ⋯ | 1.43 | <0.5 | 2560 | ⋯ | as above but with Douglas sand #1 |
| | 0.075 | 73 | ⋯ | 1.80 | <0.5 | 2560 | ⋯ | as above |
| | 0.036 | 159 | ⋯ | 2.04 | <0.5 | 11,340 | ⋯ | as above, but with lead shot grains |
| | 0.489 | 577 | ⋯ | 5.03 | <0.5 | 7,830 | ⋯ | as above, but with steel shot grains |
| | 0.170 | 0.63 | ⋯ | 0.24 | <0.5 | 2640 | ⋯ | convergent-walled flume; plane bed; sidewall correction implicit; oil fluid medium; fully laminar flow |
| | 0.103 | 2.4 | ⋯ | 0.70 | <0.5 | 2640 | ⋯ | as above |
| | 0.063 | 11 | ⋯ | 2.29 | <0.5 | 2640 | ⋯ | as above |
| | 0.066 | 7.8 | ⋯ | 1.80 | <0.5 | 2640 | ⋯ | as above |
| | 0.148 | 1.2 | ⋯ | 0.37 | <0.5 | 2950 | ⋯ | as above, but with glass beads |
| | 0.368 | 0.91 | ⋯ | 0.52 | <0.5 | 1050 | ⋯ | as above, but with styrene grains |
| | 0.387 | 2.2 | ⋯ | 0.91 | <0.5 | 1050 | ⋯ | as above |
| | 0.104 | 2.7 | ⋯ | 0.70 | <0.5 | 3100 | ⋯ | as above, but with taconite grains |

**Table 1b.** (Continued)

| | | Laboratory Mixture Grain Size Distribution | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Source | Note* | $\tau^*_{c_{v50m}}$ | $Re^*_c$† | Proposed $\tau^*_c$ Function Other Than Shields' | $D_{50m}$, mm | $\sigma_{gm}$ ($\phi$) | $\rho_s$, kg/m$^3$ | $D_{50m}/h_c$‡ | Experimental Conditions |
| | | 0.061 | 8.4 | ··· | 1.80 | <0.5 | 3100 | ··· | as above |
| | | 0.051 | 11 | ··· | 2.29 | <0.5 | 3100 | ··· | as above |
| | | 0.114 | 2.6 | ··· | 0.70 | <0.5 | 2710 | ··· | as above, but with Douglas sand #2 |
| | | 0.065 | 7.9 | ··· | 1.80 | <0.5 | 2710 | ··· | as above |
| | | 0.081 | 6.1 | ··· | 1.43 | <0.5 | 2560 | ··· | as above, but with Douglas sand #1 |
| | | 0.620 | 9.7 | ··· | 3.05 | <0.5 | 930 | ··· | as above, but with polyethylene grains |
| *White* [1970] | | 0.053†† | 36 | ··· | 2.2 | ~u | 1050 | <0.2 | straight, rectangular flume; plane bed; sidewall correction; polystyrene grains |
| | | 0.037†† | 53 | ··· | 2 | ~u | 1540 | <0.2 | as above, but with PVC grains |
| | | 0.055†† | 1.7 | ··· | 0.137 | ~u | 2600 | <0.2 | as above, but with glass ballotini grains |
| | | 0.073†† | 0.94 | ··· | 0.088 | ~u | 2600 | <0.2 | as above |
| | | 0.055†† | 2.2 | ··· | 0.17 | ~u | 2520 | <0.2 | as above, but with natural grains |
| | | 0.058†† | 1.9 | ··· | 0.153 | ~u | 2520 | <0.2 | as above |
| | | 0.071†† | 1.6 | ··· | 0.133 | ~u | 2520 | <0.2 | as above |
| | | 0.066†† | 0.92 | ··· | 0.093 | ~u | 2520 | <0.2 | as above |
| | | 0.073†† | 0.73 | ··· | 0.077 | ~u | 2520 | <0.2 | as above |
| | | 0.125†† | 0.42 | ··· | 0.044 | ~u | 2520 | <0.2 | as above |
| | | 0.102†† | 0.21 | ··· | 0.030 | ~u | 2520 | <0.2 | as above |
| | | 0.103†† | 0.20 | ··· | 0.029 | ~u | 2520 | <0.2 | as above |
| | | 0.146†† | 0.23 | ··· | 0.028 | ~u | 2520 | <0.2 | as above |
| | | 0.110†† | 0.16 | ··· | 0.024 | ~u | 2520 | <0.2 | as above |
| | | 0.112†† | 0.26 | ··· | 0.033 | ~u | 2550 | <0.2 | as above, but with crushed silica grains |
| | | 0.151†† | 0.10 | ··· | 0.016 | ~u | 2550 | <0.2 | as above |
| | | 0.037†† | 16 | ··· | 2.2 | ~u | 1050 | <0.2 | as above, but with polystyrene grains in oil; fully laminar flow (?) |
| | | 0.034†† | 40 | ··· | 2 | ~u | 1540 | <0.2 | as above, but with PVC grains |
| | | 0.143†† | 0.20 | ··· | 0.088 | ~u | 2600 | <0.2 | as above, but with glass ballotini grains |
| | | 0.132†† | 0.32 | ··· | 0.133 | ~u | 2520 | <0.2 | as above, but with natural grains |
| | | 0.122†† | 0.19 | ··· | 0.093 | ~u | 2520 | <0.2 | as above |
| | | 0.166†† | 0.16 | ··· | 0.077 | ~u | 2520 | <0.2 | as above |
| | | 0.218†† | 0.086 | ··· | 0.046 | ~u | 2520 | <0.2 | as above |
| | | 0.254†† | 0.055 | ··· | 0.033 | ~u | 2520 | <0.2 | as above |
| | | 0.288†† | 0.048 | ··· | 0.030 | ~u | 2520 | <0.2 | as above |
| | | 0.219†† | 0.033 | ··· | 0.025 | ~u | 2520 | <0.2 | as above |
| *Grass* [1970]‖‖ | 26 | 0.141†† (0.174) | 0.87 (0.97) | ··· | 0.090 | <0.24 | 2650 | <0.02? | straight, rectangular flume; plane bed; sidewall correction implicit |
| | | 0.131†† (0.154) | 0.84 (0.91) | ··· | 0.090 | <0.24 | 2650 | <0.02? | as above |
| | | 0.110†† (0.131) | 1.6 (1.7) | ··· | 0.115 | <0.13 | 2650 | <0.02? | as above |
| | | 0.086†† (0.095) | 1.8 (1.9) | ··· | 0.138 | <0.13 | 2650 | <0.03? | as above |
| | | 0.069†† (0.093) | 2.1 (2.5) | ··· | 0.165 | <0.13 | 2650 | <0.03? | as above |
| | | 0.072†† (0.079) | 2.8 (2.9) | ··· | 0.195 | <0.11 | 2650 | <0.04? | as above |
| | | 0.058 (0.091) | 1.6 (2.0) | ··· | 0.143 | <0.74 | 2650 | <0.03? | as above |
| *Sternberg* [1971]‖‖, ¶¶ | | 0.094 | 5.9 | ··· | 0.42 | 1.09 | ··· | $<2 \times 10^{-5}$ | sandy, rippled, marine channel (Pickering Passage); form drag correction similar to that in note 3 |
| | | 0.048 | 6.2 | ··· | 0.50 | 1.0 | ··· | $<2 \times 10^{-5}$ | as above |
| | | 0.038 | 5.5 | ··· | 0.50 | 1.0 | ··· | $<2 \times 10^{-5}$ | as above |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 0.041 | 3.1 | · · · | 0.33 | 1.12 | · · · | ≤0.20 | sandy marine channel (Clovis Passage) with random roughness elements; form drag correction similar to that in note 3 |
| *Everts* [1973]‖‖ | | 0.023†† | 145 | · · · | 3.57 | <0.25? | 2650 | 0.06 | straight, rectangular flume; plane bed; *Williams* [1970] sidewall correction |
| | | 0.027†† | 157 | · · · | 3.57 | <0.25? | 2650 | 0.08 | as above |
| | | 0.029†† | 162 | · · · | 3.57 | <0.25? | 2650 | 0.19 | as above |
| | | 0.023†† | 51 | · · · | 1.79 | <0.25? | 2650 | 0.03 | as above |
| | | 0.025†† | 54 | · · · | 1.79 | <0.25? | 2650 | 0.04 | as above |
| | | 0.019†† | 47 | · · · | 1.79 | <0.25? | 2650 | 0.06 | as above |
| | | 0.017†† | 44 | · · · | 1.79 | <0.25? | 2650 | 0.08 | as above |
| | | 0.017†† | 16 | · · · | 0.895 | <0.25? | 2650 | 0.01 | as above |
| | | 0.021†† | 17 | · · · | 0.895 | <0.25? | 2650 | 0.02 | as above |
| | | 0.018†† | 16 | · · · | 0.895 | <0.25? | 2650 | 0.02 | as above |
| | | 0.020†† | 17 | · · · | 0.895 | <0.25? | 2650 | 0.04 | as above |
| | | 0.020†† | 7.2 | · · · | 0.508 | <0.25? | 2650 | 0.006 | as above |
| | | 0.024†† | 8.0 | · · · | 0.508 | <0.25? | 2650 | 0.009 | as above |
| | | 0.022†† | 7.7 | · · · | 0.508 | <0.25? | 2650 | 0.01 | as above |
| | | 0.025†† | 8.1 | · · · | 0.508 | <0.25? | 2650 | 0.02 | as above |
| | | 0.029†† | 5.2 | · · · | 0.359 | <0.25? | 2650 | 0.005 | as above |
| | | 0.026†† | 4.9 | · · · | 0.359 | <0.25? | 3650 | 0.006 | as above |
| | | 0.027†† | 5.1 | · · · | 0.359 | <0.25? | 2650 | 0.009 | as above |
| | | 0.023†† | 4.6 | · · · | 0.359 | <0.25? | 2650 | 0.04 | as above |
| | | 0.032†† | 3.3 | · · · | 0.254 | <0.25? | 2650 | 0.003 | as above |
| | | 0.035†† | 3.4 | · · · | 0.254 | <0.25? | 2650 | 0.004 | as above |
| | | 0.041†† | 3.7 | · · · | 0.254 | <0.25? | 2650 | 0.008 | as above |
| | | 0.039†† | 2.1 | · · · | 0.18 | <0.25? | 2650 | 0.002 | as above |
| | | 0.046†† | 2.3 | · · · | 0.18 | <0.25? | 2650 | 0.005 | as above |
| | | 0.052†† | 1.5 | · · · | 0.127 | <0.25? | 2650 | 0.002 | as above |
| | | 0.052†† | 1.5 | · · · | 0.127 | <0.25? | 2650 | 0.002 | as above |
| | | 0.040†† | 1.3 | · · · | 0.127 | <0.25? | 2650 | 0.01 | as above |
| | | 0.056†† | 3.9 | · · · | 0.18 | <0.25? | 4700 | 0.003 | as above, but with ilmenite grains |
| | | 0.056†† | 3.9 | · · · | 0.18 | <0.25? | 4700 | 0.005 | as above |
| | | 0.062†† | 4.1 | · · · | 0.18 | <0.25? | 4700 | 0.006 | as above |
| | | 0.058†† | 2.3 | · · · | 0.127 | <0.25? | 4700 | 0.002 | as above |
| | | 0.060†† | 2.4 | · · · | 0.127 | <0.25? | 4700 | 0.002 | as above |
| | | 0.059†† | 2.4 | · · · | 0.127 | <0.25? | 4700 | 0.003 | as above |
| | | 0.063†† | 2.4 | · · · | 0.127 | <0.25? | 4700 | 0.004 | as above |
| | | 0.057†† | 1.4 | · · · | 0.09 | <0.25? | 4700 | 0.001 | as above |
| | | 0.070†† | 1.5 | · · · | 0.09 | <0.25? | 4700 | 0.002 | as above |
| | | 0.058†† | 1.4 | · · · | 0.09 | <0.25? | 4700 | 0.002 | as above |
| | | 0.081†† | 1.6 | · · · | 0.09 | <0.25? | 4700 | 0.004 | as above |
| *Fernandez Luque and van Beek* [1976]‖‖ | 28 | 0.039†† | 18 | · · · | 0.9 | 0.25 | 2640 | 0.01 | straight, rectangular flume; plane bed; sidewall correction implicit; bed slope of 0° |
| | | 0.038†† | 48 | · · · | 1.8 | 0.25 | 2640 | 0.02 | as above |
| | | 0.047†† | 127 | · · · | 3.3 | 0.25 | 2640 | 0.04 | as above |
| | | 0.043†† | 74 | · · · | 1.8 | 0.25 | 4580 | 0.02 | as above, but with magnetite grains |
| | | 0.039†† | 16 | · · · | 1.5 | 0.25 | 1340 | 0.01 | as above, but with walnut (shell?) grains |
| | | 0.031 | 16 | · · · | 0.9 | 0.25 | 2640 | 0.01 | straight, rectangular flume; plane bed; sidewall correction implicit; bed slope is 12° |
| | | 0.030 | 43 | · · · | 1.8 | 0.25 | 2640 | 0.02 | as above |
| | | 0.037 | 114 | · · · | 3.3 | 0.25 | 2640 | 0.04 | as above |

**Table 1b.** (Continued)

| Source | Note* | $\tau^*_{c_{v50m}}$ | $Re^*_c$† | Proposed $\tau^*_c$ Function Other Than Shields' | $D_{50m}$, mm | $\sigma_{gm}$ ($\phi$) | $\rho_s$, kg/m$^3$ | $D_{50m}/h_c$‡ | Experimental Conditions |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | Laboratory Mixture Grain Size Distribution | | | | |
| | | 0.035 | 67 | ··· | 1.8 | 0.25 | 4580 | 0.02 | as above, but with magnetite grains |
| | | 0.031 | 14 | ··· | 1.5 | 0.25 | 1340 | 0.01 | as above, but with walnut (shell?) grains |
| | | 0.027 | 14 | ··· | 0.9 | 0.25 | 2640 | 0.01 | straight, rectangular flume; plane bed; sidewall correction implicit; bed slope is 18° |
| | | 0.026 | 39 | ··· | 1.8 | 0.25 | 2640 | 0.02 | as above |
| | | 0.031 | 112 | ··· | 3.3 | 0.25 | 2640 | 0.04 | as above |
| | | 0.028 | 59 | ··· | 1.8 | 0.25 | 4580 | 0.02 | as above, but with magnetite grains |
| | | 0.026 | 13 | ··· | 1.5 | 0.25 | 1340 | 0.01 | as above, but with walnut (shell?) grains |
| | | 0.025 | 14 | ··· | 0.9 | 0.25 | 2640 | 0.01 | straight, rectangular flume; plane bed; sidewall correction implicit; bed slope is 22° |
| | | 0.021 | 36 | ··· | 1.8 | 0.25 | 2640 | 0.02 | as above |
| | | 0.027 | 99 | ··· | 3.3 | 0.25 | 2640 | 0.04 | as above |
| | | 0.023 | 55 | ··· | 1.8 | 0.25 | 4580 | 0.02 | as above, but with magnetite grains |
| | | 0.024 | 12 | ··· | 1.5 | 0.25 | 1340 | 0.01 | as above, but with walnut (shell?) grains |
| From *Mantz* [1977] | | | | | | | | | |
| *Schoklitsch* [1914]** | | 0.066 | 176 | ··· | 3.06 | 0 | 2700 | ··· | straight, rectangular flume; plane bed; slate grains; no sidewall correction (?) |
| | | 0.041 | 37 | ··· | 1.24 | 0 | 2700 | ··· | as above |
| *Ho* [1939]** | | 0.018 | 265 | ··· | 6.71 | 0.56 | 2660 | ≤0.17 | straight, rectangular flume; plane bed; slate grains; no sidewall correction (?) |
| | | 0.025†† | 225 | ··· | 5.71 | 0.46 | 2660 | ≤0.12 | as above |
| | | 0.021 | 102 | ··· | 3.26 | 1.13 | 2700 | ≤0.09 | as above |
| | | 0.034 | 65 | ··· | 2.21 | 0.90 | 2490 | ≤0.20 | as above |
| | | 0.024 | 33 | ··· | 1.63 | 0.98 | 2450 | ≤0.05 | as above |
| *Mantz* [1975] | | 0.094†† | 0.70 | ··· | 0.066 | ~0.20 | 2650 | 0.0007 | straight, rectangular flume; plane bed; *Shimizu* [1989] sidewall correction;‡‡ quartz grains |
| | | 0.121†† | 0.79 | ··· | 0.066 | ~0.20 | 2650 | 0.0008 | as above |
| | | 0.127†† | 0.81 | ··· | 0.066 | ~0.20 | 2650 | 0.001 | as above |
| | | 0.122†† | 0.79 | ··· | 0.066 | ~0.20 | 2650 | 0.002 | as above |
| | | 0.126†† | 0.81 | ··· | 0.066 | ~0.20 | 2650 | 0.003 | as above |
| | | 0.155†† | 0.48 | ··· | 0.045 | ~0.20 | 2650 | 0.0004 | as above |
| | | 0.147†† | 0.47 | ··· | 0.045 | ~0.20 | 2650 | 0.0006 | as above |
| | | 0.134†† | 0.45 | ··· | 0.045 | ~0.20 | 2650 | 0.0008 | as above |
| | | 0.127†† | 0.44 | ··· | 0.045 | ~0.20 | 2650 | 0.001 | as above |
| | | 0.140†† | 0.46 | ··· | 0.045 | ~0.20 | 2650 | 0.002 | as above |
| | | 0.140†† | 0.27 | ··· | 0.030 | ~0.20 | 2650 | 0.0003 | as above |
| | | 0.133†† | 0.27 | ··· | 0.030 | ~0.20 | 2650 | 0.0004 | as above |
| | | 0.164†† | 0.29 | ··· | 0.030 | ~0.20 | 2650 | 0.0005 | as above |
| | | 0.161†† | 0.29 | ··· | 0.030 | ~0.20 | 2650 | 0.0008 | as above |
| | | 0.165†† | 0.28 | ··· | 0.030 | ~0.20 | 2650 | 0.001 | as above |
| | | 0.198†† | 0.12 | ··· | 0.015 | ~0.20 | 2650 | 0.0002 | as above |
| | | 0.070†† | 0.77 | ··· | 0.076 | ~0.2? | 2740 | ≤0.004 | as above, but with micaceous flakes** and no sidewall correction |
| | | 0.062†† | 0.71 | ··· | 0.076 | ~0.2? | 2740 | ≤0.004 | as above |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 0.055†† | 0.39 | ⋯ | 0.053 | ~0.2? | 2740 | ≤0.003 | as above |
| | | 0.070†† | 0.44 | ⋯ | 0.053 | ~0.2? | 2740 | ≤0.003 | as above |
| | | 0.061†† | 0.29 | ⋯ | 0.042 | ~0.2? | 2740 | ≤0.002 | as above |
| | | 0.068†† | 0.30 | ⋯ | 0.042 | ~0.2? | 2740 | ≤0.002 | as above |
| | | 0.063†† | 0.20 | ⋯ | 0.033 | ~0.2? | 2740 | ≤0.002 | as above |
| | | 0.076†† | 0.22 | ⋯ | 0.033 | ~0.2? | 2740 | ≤0.002 | as above |
| | | 0.092†† | 0.15 | ⋯ | 0.024 | ≥0.2? | 2740 | ≤0.001 | as above |
| | | 0.087†† | 0.15 | ⋯ | 0.024 | ~0.2? | 2740 | ≤0.001 | as above |
| | | 0.140†† | 0.10 | ⋯ | 0.016 | ~0.2? | 2740 | ≤0.001 | as above |
| *Wimbush and Lesht* [1979]‖‖, ¶¶ | 27 | 0.030 | 2.4 | ⋯ | 0.35 | ≤0.84 | 2930 (?)–1100 | $5 \times 10^{-7}$ | sandy, rippled, marine channel (Straights of Florida); form drag correction similar to that in note 3 |
| *Yalin and Karahan* [1979] | | 0.038†† | 25 | ⋯ | 1.00 | <0.34–<0.54? | 2650 | 0.16 | straight, rectangular flume; plane bed; sidewall correction implicit? |
| | | 0.030†† | 9.4 | ⋯ | 0.56 | <0.34–<0.54? | 2650 | 0.12 | as above |
| | | 0.113†† | 1.5 | ⋯ | 0.10 | <0.34–<0.54? | 2650 | 0.02 | as above |
| | | 0.036†† | 6.1 | ⋯ | 0.40 | <0.34–<0.54? | 2650 | 0.09 | as above |
| | | 0.053†† | 2.5 | ⋯ | 0.19 | <0.34–<0.54? | 2650 | 0.03 | as above |
| | | 0.079†† | 1.8 | ⋯ | 0.14 | 0 | 2500 | 0.03 | as above, but with glass bead grains |
| | | 0.178†† | 0.13 | ⋯ | 1.00 | <0.34–<0.54 | 2650 | 0.09 | as above, but with natural grains in a water-glycerin mixture and fully laminar flow |
| | | 0.156†† | 0.30 | ⋯ | 1.88 | <0.34–<0.54 | 2650 | 0.24 | as above |
| | | 0.135†† | 0.55 | ⋯ | 2.86 | <0.34–<0.54 | 2650 | 0.38 | as above |
| | | 0.172†† | 0.054 | ⋯ | 0.56 | <0.34–<0.54 | 2650 | 0.04 | as above |
| | | 0.110†† | 0.78 | ⋯ | 1.00 | <0.34–<0.54 | 2650 | 0.10 | as above |
| | | 0.092†† | 1.9 | ⋯ | 1.88 | <0.34–<0.54 | 2650 | 0.30 | as above |
| | | 0.141†† | 0.38 | ⋯ | 0.56 | <0.34–<0.54 | 2650 | 0.04 | as above |
| | | 0.086†† | 3.4 | ⋯ | 2.86 | <0.34–<0.54 | 2650 | 0.41 | as above |
| | | 0.091†† | 3.4 | ⋯ | 2.86 | <0.34–<0.54 | 2650 | 0.25 | as above |
| | | 0.134†† | 0.86 | ⋯ | 1.00 | <0.34–<0.54 | 2650 | 0.07 | as above |
| | | 0.143†† | 0.38 | ⋯ | 0.56 | <0.34–<0.54 | 2650 | 0.03 | as above |
| | | 0.110†† | 2.0 | ⋯ | 1.88 | <0.34–<0.54 | 2650 | 0.16 | as above |
| | | 0.140†† | 0.80 | ⋯ | 0.56 | <0.34–<0.54 | 2650 | 0.04 | as above |
| | | 0.069†† | 6.4 | ⋯ | 2.86 | <0.34–<0.54 | 2650 | 0.31 | as above |
| | | 0.086†† | 3.8 | ⋯ | 1.88 | <0.34–<0.54 | 2650 | 0.18 | as above |
| | | 0.106†† | 1.6 | ⋯ | 1.00 | <0.34–<0.54 | 2650 | 0.09 | as above |
| *Ikeda* [1982] | | 0.020†† | 8.7 | ⋯ | 0.42 | 0.36 | ⋯ | 0.006 | laterally tilting wind tunnel; straight, rectangular walls; plane bed; *Adachi* [1962] sidewall correction |
| | | 0.047†† | 72 | ⋯ | 1.3 | 0.31 | ⋯ | 0.02 | as above |
| *Young and Mann* [1985]¶¶ | 28 | 0.030 | 2.5 | ⋯ | 0.36 | 1.8 | 1500 | $1.5 \times 10^{-5}$ | straight, rectangular sea flume; plane bed or low-amplitude bed forms; sidewall correction implicit; skeletal carbonate grains |
| | | 0.028 | 4.3 | ⋯ | 0.53 | >2.1 | 1500 | $2.3 \times 10^{-5}$ | as above |
| | | 0.109 | 1.6 | ⋯ | 0.18 | <2.0 | 1500 | $1.4 \times 10^{-5}$ | as above |
| | | 0.075 | 1.5 | ⋯ | 0.19 | <2.1 | 1500 | $7.3 \times 10^{-6}$ | as above |
| | | 0.026 | 0.58 | ⋯ | 0.14 | <2.0 | 1500 | $5.3 \times 10^{-6}$ | as above |
| | | 0.105 | 1.4 | ⋯ | 0.16 | <2.1 | 1500 | $5.3 \times 10^{-6}$ | as above |
| | | 0.059 | 2.0 | ⋯ | 0.25 | <2.2 | 1500 | $1.0 \times 10^{-5}$ | as above |
| | | 0.070 | 3.6 | ⋯ | 0.35 | 2.0 | 1500 | $1.5 \times 10^{-5}$ | as above |
| | | 0.047 | 3.0 | ⋯ | 0.35 | 2.2 | 1500 | $2.3 \times 10^{-5}$ | as above |

**Table 1b.** (Continued)

| | | Laboratory Mixture Grain Size Distribution | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Source | Note* | $\tau^*_{c_{v50m}}$ | $Re^*_c$† | Proposed $\tau^*_c$ Function Other Than Shields' | $D_{50m}$, mm | $\sigma_{gm}$ ($\phi$) | $\rho_s$, kg/m$^3$ | $D_{50m}/h_c$‡ | Experimental Conditions |
| *Prager et al.* [1996]¶¶ | | 0.025†† | 7.1 | ... | 0.5 | ≤0.5? | 2660 | 0.04 | straight, rectangular flume; plane bed; *Shimizu* [1989] sidewall correction;‡‡ well-rounded quartz grains |
| | | 0.021†† | 6.6 | ... | 0.5 | ≤0.5? | 2770 | 0.04 | as above, but with ooid grains |
| | | 0.025†† | 7.3 | ... | 0.5 | ≤0.5? | 2730 | 0.04 | as above, but with rounded, mixed carbonate and terrigenous grains |
| | | 0.022†† | 6.2 | ... | 0.5 | ≤0.5? | 2500 | 0.04 | as above, but with platy, skeletal carbonate grains |
| | | 0.020 | 11 | ... | 0.7 | 0.80 | 2700? | 0.05 | as above |
| | | 0.017 | 19 | ... | 1.1 | 0.93 | 2670? | 0.09 | as above |

Note that symbols for similar footnotes may be different in Tables 1a–1e. See notation section for symbols not previously defined in text. See respective appendix notes for values in parentheses. Here "u" denotes uniform grain sizes ($\sigma_g \leq 0.5$) and "m" denotes mixed grain sizes ($\sigma_g > 0.5$), where $\sigma_g$ is the graphic standard deviation defined as $(\phi_{84} - \phi_{16})/2$ [*Folk*, 1974].

*See appendix.

†$Re^*_c = u^*_c D_{50}/\nu$. Where unreported by a source, $Re^*_c$ values are back-calculated from available data (see footnote keyed to †, Table 1a).

‡Where unreported by a source, $h_c$ values are back-calculated from critical depth-slope products using reported data (see footnote keyed to ‡, Table 1a).

§Here we describe grain protrusion in terms of projection and exposure [sensu *Kirchner et al.*, 1990].

||Sidewall correction for the difference in wall and bed grain roughness but not for the proximity of walls (i.e., *W*/*h*).

¶Use of the average velocity in the *Einstein* [1942; 1950], *Johnson* [1942] and *Vanoni and Brooks* [1957] equations likely overestimates $\tau'$ (see note 3).

**Reported data are with respect to the mean nominal grain diameter. Nominal diameters are assumed equivalent to intermediate grain diameters [*Cui and Komar*, 1984]. Mean and median sizes are similar for near-uniform sediment.

††Used in Plate 1.

‡‡Sidewall correction applied by current authors. *Shimizu*'s [1989] correction accounts for both the difference in wall and bed grain roughness and the proximity of walls (i.e., *W*/*h*). We assumed a bed grain roughness 100 times greater than wall roughness for smooth flume walls.

§§Reported data are with respect to the geometric mean grain size.

||||Reported data are with respect to mean grain sizes. Mean and median grain sizes are assumed similar for near-uniform sediment.

¶¶Reported data are determined from bulk (i.e., surface and subsurface mixture) grain size sampling, treated here as mixture-based values.

**Table 1c.** Previously Reported $\tau^*_{c_{50}}$ Values: Competence (Largest Mobile Grain)

| Source | Note* | $\tau^*_{c_{q50s}}$ | $Re^*_c$† | Proposed $\tau^*_c$ Function Other Than Shields' | Surface Grain Size Distribution: $D_{50s}$, mm | $\sigma_{gs}$ ($\phi$) | $\rho_s$, kg/m$^3$ | $D_{50s}/h_c$‡ | Experimental Conditions |
|---|---|---|---|---|---|---|---|---|---|
| *Carling* [1983] | 29 | 0.020 | 6,629 | $\tau^*_{c_{qi}} = 1.17 Re_i^{*-0.46}$ | 62 | ~2 | 2710 | 0.29 | natural, steep, gravel-bedded channel (Great Eggleshope Beck); coarse-grained plane bed/alternate bar morphology (?); no form drag or sidewall correction |
| | | 0.080 | 18,634 | $\tau^*_{c_{qi}} = 4.99 Re_i^{*-0.42}$ | 77 | ~1.7 | 2460 | 0.34 | natural, steep, narrow, gravel-bedded channel (Carl Beck); coarse-grained plane bed morphology (?); no form drag or sidewall correction |
| *Hammond et al.* [1984] | 30 | 0.025§ | 877 | $\tau^*_{c_{qi}} = 0.025(D_i/D_{50s})^{-0.60}$ | 15.5 | 0.90 | 2650 | ≪0.20 | natural, planar, tidal channel (West Solent) |
| *Andrews and Erman* [1986] | 31 | 0.050§ | 8,377 | $\tau^*_{c_{qi}} = 0.101(D_i/D_{50ss})^{-1.07}$ | 58 | 0.97 | ··· | 0.14 | natural, meandering, pool-riffle channel (Sagehen Creek); no form drag or sidewall correction |
| From *Komar* [1987a] | 32 | | | | | | | | |
| *Milhous* [1973] | 33 | 0.027§ | 7,277 | $\tau^*_{c_{qi}} = 0.044(D_i/D_{50ss})^{-0.43}$ | 63 | <0.71 | 2850 | 0.20 | same as first entry of Table 1a |
| *Milhous* [1973] | 33 | 0.024 | 6,861 | $\tau^*_{c_{qi}} = 0.044(D_i/D_{50ss})^{-0.53}$ | 63 | <0.71 | 2850 | 0.23 | as above |
| *Carling* [1983] | 34 | 0.021 | 6,738 | $\tau^*_{c_{qi}} = 0.045(D_i/D_{50})^{-0.68}$ | 62 | ~2 | 2710 | 0.28 | same as *Carling* [1983], first entry of Table 1c (Great Eggleshope Beck) |
| *Carling* [1983] | 34 | 0.022 | 6,896 | $\tau^*_{c_{qi}} = 0.045(D_i/D_{50})^{-0.64}$ | 62 | ~2 | 2710 | 0.27 | as above |
| *Hammond et al.* [1984] | 35 | 0.027§ | 911 | $\tau^*_{c_{qi}} = 0.045(D_i/D_{50})^{-0.71}$ | 15.5 | 0.90 | 2650 | ≪0.20 | same as *Hammond et al.* [1984], second entry of Table 1c |
| From *Komar* [1987b] | | | | | | | | | |
| *Fahnestock* [1963] | 36 | 0.031 | 21,033 | ··· | 134 | 1.79 | ··· | 0.61 | natural, proglacial, braided channel (White River); no form drag or sidewall correction |
| *Ferguson et al.* [1989] | | 0.047§ | 37,880 | $\tau^*_{c_{qi}} = 0.047(D_i/D_{50s})^{-0.88}$ | 73 | ··· | 2800 | 0.20 | natural, proglacial, braided, gravel channel (White River); form drag and sidewall correction as in note 3 |
| From *Komar and Carling* [1991] | 37 | | | | | | | | |
| *Milhous* [1973] | | 0.028 | 7,601 | $\tau^*_{c_{qi}} = 0.059(D_i/D_{50ss})^{-0.64}$ | 63 | <0.71 | 2850 | 0.27 | same as first entry of Table 1a |
| *Komar and Carling* [1991] | | 0.039§ | 9,182 | $\tau^*_{c_{qi}} = 0.039(D_i/D_{50s})^{-0.82}$ | 62 | ~2 | 2710 | 0.15 | same as *Carling* [1983] from *Komar* [1987a] |
| *Ashworth et al.* [1992] | | 0.049§ | 1,807 | $\tau^*_{c_{qi}} = 0.049(D_i/D_{50s})^{-0.69}$ | 21 | ··· | 2650 | 0.07 | same as *Ashworth et al.* [1992], sixth entry of Table 1a |
| *Lepp et al.* [1993] | 38 | 0.149 | 28,421 | $\tau^*_{c_{qi}} = 0.149(D_i/D_{50s})^{-0.40}$ | 91 | 0.40 | ··· | 0.35 | Natural, steep, gravel-bedded river; bed form type not reported; *Shimizu* [1989] sidewall correction,‖ but no form drag correction |
| | | 0.143 | 24,693 | $\tau^*_{c_{qi}} = 0.143(D_i/D_{50s})^{-0.50}$ | 84 | 0.33 | ··· | 0.45 | as above |
| | | 0.155 | 40,645 | $\tau^*_{c_{qi}} = 0.155(D_i/D_{50s})^{-0.47}$ | 114 | 0.38 | ··· | 0.48 | as above |

**Table 1c.** (Continued)

| Source | Note* | $\tau^*_{c_{q50s}}$ | $Re^*_c$† | Proposed $\tau^*_c$ Function Other Than Shields' | $D_{50s}$, mm | $\sigma_{gs}$ ($\phi$) | $\rho_s$, kg/m$^3$ | $D_{50s}/h_c$‡ | Experimental Conditions |
|---|---|---|---|---|---|---|---|---|---|
| | | | | **Surface Grain Size Distribution** | | | | | |
| *Ferguson* [1994] | | 0.074 | 14,096 | $\tau^*_{c_{qi}} = 0.074(D_i/D_{50s})^{-0.87}$ | 72 | 1.56 | 2650 | 0.38 | natural boulder-bed stream (Roaring River); bed form type not reported; no sidewall correction; *Thompson and Campbell* [1979] form drag correction for relative roughness |
| | | 0.078 | 16,005 | $\tau^*_{c_{qi}} = 0.078(D_i/D_{50s})^{-0.88}$ | 77 | 1.14 | 2650 | 0.40 | as above |
| | | 0.061 | 34,701 | $\tau^*_{c_{qi}} = 0.061(D_i/D_{50s})^{-0.89}$ | 140? | 1.06? | 2650 | 0.47? | as above |
| | | 0.070 | 24.606 | $\tau^*_{c_{qi}} = 0.070(D_i/D_{50s})^{-0.78}$ | 106 | 1.06 | 2650 | 0.31 | as above |
| | | 0.047§ | 11,943 | $\tau^*_{c_{qi}} = 0.047(D_i/D_{50s})^{-0.69}$ | 75 | ··· | 2650 | 0.03 | as above, but for the Gaula River |
| *Wathen et al*. [1995] | | 0.059§ | 2,025 | $\tau^*_{c_{qi}} = 0.059(D_i/D_{50s})^{-0.70}$ | 21.3 | ~1.6 | 2650 | 0.02–0.21 | same as *Wathen et al*. [1995], tenth entry of Table 1a |

| Source | Note* | $\tau^*_{c_{q50ss}}$ | $Re^*_c$† | Proposed $\tau^*_c$ Function Other Than Shields' | $D_{50ss}$, mm | $\sigma_{gss}$ ($\phi$) | $\rho_s$, kg/m$^3$ | $D_{50ss}/h_c$‡ | Experimental Conditions |
|---|---|---|---|---|---|---|---|---|---|
| | | | | **Subsurface Grain Size Distribution** | | | | | |
| *Andrews and Erman* [1986] | | 0.101 | 4,429 | $\tau^*_{c_{qi}} = 0.101(D_i/D_{50ss})^{-1.07}$ | 30 | 2.25 | ··· | 0.07 | same as *Andrews and Erman* [1986], third entry of Table 1c |
| From *Komar* [1987a] | | | | | | | | | |
| *Milhous* [1973] | | 0.044 | 1,662 | $\tau^*_{c_{qi}} = 0.044(D_i/D_{50ss})^{-0.43}$ | 20 | <2.67 | 2850 | 0.12 | same as *Parker and Klingeman* [1982], first entry of Table 1a |
| *Carling* [1983] | 39 | 0.037 | 2,570 | $\tau^*_{c_{qi}} = 0.045(D_i/D_{50})^{-0.68}$ | 27 | ··· | 2710 | 0.16 | same as *Carling* [1983], first entry of Table 1c |
| From *Komar and Carling* [1991] | | | | | | | | | |
| *Milhous* [1973] | | 0.059 | 1,974 | $\tau^*_{c_{qi}} = 0.059(D_i/D_{50ss})^{-0.64}$ | 20 | <2.67 | 2850 | 0.13 | same as *Parker and Klingeman* [1982], first entry of Table 1a |
| *Komar and Carling* [1991] | | 0.077 | 3,707 | $\tau^*_{c_{qi}} = 0.039(D_i/D_{50s})^{-0.82}$ | 27 | ··· | 2710 | 0.08 | same as *Carling* [1983], first entry of Table 1c |

Note that symbols for similar footnotes may be different in Tables 1a–1e. See notation section for symbols not previously defined in text.

*See appendix.

†$Re^*_c = u^*_c D_{50}/\nu$. Where unreported by a source, $Re^*_c$ values are back-calculated from available data (see footnote keyed to †, Table 1a).

‡Where unreported by a source, $h_c$ values are back-calculated from critical depth-slope products using reported data (see footnote keyed to ‡, Table 1a).

§Used in Plate 1.

‖Sidewall correction applied by current authors. *Shimizu*'s [1989] correction accounts for both the difference in wall and bed grain roughness and the proximity of walls (i.e., $W/h$). We assumed a bed grain roughness 100 times greater than wall roughness for smooth flume walls.

**Table 1d.** Previously Reported $\tau^*_{c_{50}}$ Values: Theoretical

| Source | Note* | $\tau^*_{c_{i50s}}$ | $Re^*_c$† | Proposed $\tau^*_c$ Function Other Than Shields' | Surface Grain Size Distribution: $D_{50s}$, mm | $\sigma_{gs}$ ($\phi$) | $\rho_s$, kg/m$^3$ | $D_{50s}/h_c$‡ | Experimental Conditions |
|---|---|---|---|---|---|---|---|---|---|
| *White* [1940] | 40 | 0.209 | 44 | see reference | 0.9 | ~u | 2600 | na | theoretical plane bed; single grain within like bed; variable packing, projection and exposure;§ Φ = 45° |
| | | 0.136 | 51 | see reference | 0.71 | ~u | 7900 | na | as above |
| | | 0.210 | 693 | see reference | 5.6 | ~u | 2600 | na | as above |
| | | 0.209 | 117 | see reference | 0.9 | ~u | 2100 | na | as above, but with air fluid medium |
| | | 0.208 | 1,829 | see reference | 5.6 | ~u | 2100 | na | as above |
| *Chepil* [1959] | | 0.018 | ··· | see reference | ··· | u, m | ··· | na | theoretical plane bed undergoing en masse motion; variable packing, projection and exposure;§ Φ = 24° |
| *Egiazaroff* [1965]‖ | | 0.060 | 1,000 | see reference | ··· | u, m | ··· | na | theoretical plane bed; single grain on like, uniform, or mixed-grain bed; significant projection and exposure§ |
| *Ikeda* [1982] | 41 | 0.061 (0.034) | 1,000 | see reference | ··· | 0 | ··· | na | theoretical plane bed; single grain on like, uniform bed; significant projection and exposure;§ Φ = 45° |
| *Naden* [1987] | 42 | 0.046 (0.030, 0.021) | >77 (62, 52) | see reference | >2 | 0 | 2650 | na | theoretical plane bed; single grain on like, uniform bed; full projection and exposure;§ Φ = 35° |
| | | 0.216 (0.124, 0.080) | >167 (127, 102) | see reference | >2 | 0 | 2650 | na | as above, but with two grains on like, uniform bed; full projection, but no exposure§ for grain of interest |
| | | 0.063 (0.036, 0.023) | >90 (68, 55) | see reference | >2 | 0 | 2650 | na | as above, but with single grain within like, uniform bed; no projection or exposure§ |
| *Wiberg and Smith* [1987] | | 0.060 | 1,000 | see reference | ··· | 0 | 2650 | na | theoretical plane bed; single grain on like, uniform bed; significant projection and exposure;§ Φ = 60° |
| | | 0.040 | 1,000 | see reference | ··· | 0 | 2650 | na | as above, but with Φ = 50° |
| *James* [1990] | | 0.046 | 100 | see reference | ··· | 0 | ··· | na | theoretical plane bed; single grain within bed; moderate projection and exposure;§ Φ = 64° |
| | | 0.011 | 130 | see reference | ··· | 0? | ··· | na | theoretical plane bed; single grain on like, uniform bed; significant projection and exposure;§ Φ = 19° |
| *Kirchner et al*. [1990] | 43 | 0.125 | 325–480 | see reference | 3.74–4.85 | ~0.94–1.18 | 2650 | na | theoretical plane bed; single grain on like, mixed-grain bed; distribution of friction angle, protrusion and exposure§ |
| *Buffington et al*. [1992] | 44 | 0.100 | 334–12,145 | see reference | 4.1–45 | 0.67–1.50 | 2650 | na | theoretical plane bed; single grain on like, mixed-grain bed; distribution of friction angle, protrusion and exposure§ |
| *Bridge and Bennett* [1992] | | 0.060 | 1,000 | see reference | ··· | 0 | ··· | na | theoretical plane bed with Φ ≈ 39°, protrusion of 0.5 [sensu *Fenton and Abbott*, 1977], Corey shape factor of 1, and Powers roundness of 6 single grain on like, uniform bed |

**Table 1d.** (Continued)

| | | Surface Grain Size Distribution | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Source | Note* | $\tau^*_{c_{t50s}}$ | $Re^*_c$† | Proposed $\tau^*_c$ Function Other Than Shields' | $D_{50s}$, mm | $\sigma_{gs}$ ($\phi$) | $\rho_s$, kg/m$^3$ | $D_{50s}/h_c$‡ | Experimental Conditions |
| | | 0.042 | 10,000 | see reference | ··· | 0 | ··· | na | as above |
| *Jiang and Haff* [1993] | 45 | 0.086–0.161 | 700–1,000 | see reference | 7.2 (?) | m | 2650 | na | simulated, heterogeneous, plane bed surface undergoing a "slab" shear; variable projection and exposure§ |
| *Ling* [1995] | 46 | 0.050 | 500 | see reference | ··· | 0 | ··· | na | theoretical plane bed; single grain on like, uniform bed; significant projection and exposure§ |

Note that symbols for similar footnotes may be different in Tables 1a–1e. See notation section for symbols not previously defined in text. See respective appendix notes for values in parentheses. Here "u" denotes uniform grain sizes ($\sigma_g \leq 0.5$), and "m" denotes mixed grain sizes ($\sigma_g > 0.5$), where $\sigma_g$ is the graphic standard deviation defined as $(\phi_{84} - \phi_{16})/2$ [*Folk*, 1974]; na, not applicable.

*See appendix.

†$Re^*_c = u^*_c D_{50}/\nu$. Where unreported by a source, $Re^*_c$ values are back-calculated from available data (see footnote keyed to †, Table 1a).

‡Where unreported by a source, $h_c$ values are back-calculated from critical depth-slope products using reported data (see footnote keyed to ‡, Table 1a).

§Here we describe grain protrusion in terms of projection and exposure [sensu *Kirchner et al.*, 1990].

‖Reported data are with respect to mean grain sizes. Mean and median grain sizes are assumed similar for near-uniform sediment.

**Table 1e.** Previously Reported $\tau^*_{c_{50}}$ Values: Other

| | Surface Grain Size Distribution | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Source | $\tau^*_{c_{50s}}$ | $Re^*_c$† | Proposed $\tau^*_c$ Function Other Than Shields' | $D_{50s}$, mm | $\sigma_{gs}$ ($\phi$) | $\rho_s$, kg/m$^3$ | $D_{50s}/h_c$‡ | Experimental Conditions |
| | | | *Equivalence of Bed Load and Surface Grain Size* | | | | | |
| *Çeçen and Bayazit* [1973] | 0.044 | 1,481 | ··· | 14.6 | 0.72 | ··· | 0.26 | straight, rectangular flume; plane bed; no sidewall correction |
| From *Komar and Carling* [1991] | | | | | | | | |
| *Milhous* [1973] | 0.037 | 8,738 | $\tau^*_{c_{q50l}} = 0.104(D_{50l}/D_{50ss})^{-0.89}$ | 63 | <0.71 | 2850 | 0.20 | same as *Parker and Klingeman* [1982], first entry of Table 1a |
| *Komar and Carling* [1991] | 0.055 | 10,904 | $\tau^*_{c_{q50l}} = 0.055(D_{50l}/D_{50s})^{-0.89}$ | 62 | ~2 | 2710 | 0.11 | same as *Carling* [1983], first entry of Table 1c |
| | | | *Smallest Transport Captured by Bed Load Trap* | | | | | |
| *Powell and Ashworth* [1995] | 0.010 | 2,790 | ··· | 49 | ··· | 2586 | 0.20 | natural, straight, gravel-bedded channel with low-amplitude medial bar (River Wharfe); loose, open framework, subangular grains; no sidewall or form drag correction |
| | 0.011 | 2,860 | ··· | 49 | ··· | 2586 | 0.18 | as above |
| | 0.055 | 6,700 | ··· | 49 | ··· | 2586 | 0.03 | as above, but with imbricated algal-covered grains |
| | 0.067 | 7,590 | ··· | 49 | ··· | 2586 | 0.03 | as above |

Note that symbols for similar footnotes may be different in Tables 1a–1e. See notation section for symbols not previously defined in text.

*See appendix.

†$Re^*_c = u^*_c D_{50}/\nu$. Where unreported by a source, $Re^*_c$ values are back-calculated from available data (see footnote keyed to †, Table 1a).

‡Where unreported by a source, $h_c$ values are back-calculated from critical depth-slope products using reported data (see footnote keyed to ‡, Table 1a).

*Nikuradse* [1933] sand grain roughness).

$m$ mixed grain size.

$n_g$, $n_b$ Manning roughnesses due to grains and the combined effects of grains and bed forms [*Meyer-Peter and Müller*, 1948].

$Q$, $Q_b$ total volumetric fluid discharge and that acting on the grains and bed forms [*Meyer-Peter and Müller*, 1948].

$Re^*_c$, $Re^*_{ci}$ critical grain/boundary Reynolds number for $D_{50}$ and $D_i$.

$u$ uniform grain size.

$u^*$, $u^*_c$ general and critical shear velocities.

$W/h$ width-to-depth ratio.

$\alpha$ coefficient.

$\beta$ exponent.

$\theta$ angular bed surface slope.

$\nu$ kinematic viscosity.

$\rho$, $\rho_s$ fluid and sediment densities.

$\sigma_g$, $\sigma_{gs}$, $\sigma_{gss}$, $\sigma_{gm}$ sorting coefficient (*Folk*'s [1974] graphic standard deviation) and those for the surface, subsurface, and laboratory mixtures.

$\tau_0$ total boundary shear stress.

$\tau'$, $\tau''$, ... components of total boundary shear stress due to roughness elements such as grains ($\tau'$), bed forms, walls, large woody debris, etc.

$\tau_c$ critical shear stress for incipient motion.

$\tau^*_c$ dimensionless critical shear stress for incipient motion.

$\tau^*_{c_i}$, $\tau^*_{c_{50}}$, $\tau^*_{c_{50s}}$, $\tau^*_{c_{50ss}}$, $\tau^*_{c_{50m}}$ $\tau^*_c$ of $D_i$, $D_{50}$, $D_{50s}$, $D_{50ss}$, and $D_{50m}$.

$\tau^*_{c_{qi}}$, $\tau^*_{c_{q50}}$, $\tau^*_{c_{q50s}}$, $\tau^*_{c_{q50ss}}$, $\tau^*_{c_{q50l}}$ $\tau^*_c$ of $D_i$, $D_{50}$, $D_{50s}$, $D_{50ss}$, and $D_{50l}$ based on empirical competence equations determined from coupled bed load sampling and shear stress measurement.

$\tau^*_{c_{ri}}$, $\tau^*_{c_{r50}}$, $\tau^*_{c_{r50s}}$, $\tau^*_{c_{r50m}}$ $\tau^*_c$ of $D_i$, $D_{50}$, $D_{50s}$, and $D_{50m}$ for a specified reference bed load transport rate.

$\tau^*_{c_{vi}}$, $\tau^*_{c_{v50}}$, $\tau^*_{c_{v50m}}$ $\tau^*_c$ of $D_i$, $D_{50}$, and $D_{50m}$ based on visual observation of incipient motion.

$\phi_{16}$, $\phi_{84}$ $\log_2$ grain sizes for which 16% and 84% of the grains are finer.

$\Phi$ intergranular friction angle.

## Appendix: Notes for Tables 1a–1e

1. We estimated the exponent of the *Parker and Klingeman* [1982] $\tau^*_{c_{ri}}$ function from their Figure 3. To calculate $D_{50s}/h_c$, we used a slope of 0.01 based on armor-breaching discharges (over 40 feet$^3$/s (1.13 m$^3$/s)) during the winter of 1971 [*Milhous*, 1973, Table I-3]; except where reported differently, we assumed this same slope for all sources using *Milhous*' [1973] data.

2. We calculated $\tau^*_{c_{r50s}}$ with $D_i = D_{50s} = 54$ mm [*Parker and Klingeman*, 1982] and $D_{50ss} = 19.5$ mm [*Wilcock and Southard*, 1988, Table 1].

3. $D_{50s}$ values are averages of those reported in *Ashworth and Ferguson*'s [1989] Table 1. Although *Ashworth and Ferguson* [1989] used local velocity profiles rather than depth-slope products to determine shear stress, they used the full velocity profile rather than just near-bed values. The full profile includes all local roughness effects (bed form drag, etc.) and likely overestimates the effective shear stress ($\tau'$) (see discussion of segmented velocity profiles by *Middleton and Southard* [1984] and *Smith and McLean* [1977]). Their local velocity measures do, however, implicitly account for sidewall effects.

4. We estimated $\tau^*_{c_{r50s}}$ from *Wilcock and Southard*'s [1988] Oak Creek equation using the same $D_i$ and $D_{50ss}$ values as those in note 2 but with the coefficient of the equation reduced by 55% for bed form drag [*Wilcock*, 1993].

5. We calculated $\tau^*_{c_{r50s}}$ from the Shields equation using $\tau_{c_{r50s}}$ determined from *Wilcock and McArdell*'s [1993] Figure 8 expression, with $D_{50s}$ estimated by averaging their Figure 5 data for runs 7b, 7c, 2, 4, 5, 6, and 14c (assumed to be equal to "start-up").

6. We estimated $\tau^*_{c_{r50s}}$ for runs 4 and 5 from *Day*'s [1981] Figure 2 using $D_{50s}$ values [*Day*, 1981, Figure 1] from the immediately preceding runs (i.e., 3 and 4, respectively) and recognizing that $\tau^*_c$ is the square of the Ackers-White mobility number.

7. We calculated $\tau^*_{c_{r50s}}$ from the Shields equation using $\tau_{c_{r50s}}$ regressed from particle velocity and $u^*$ data for $D_{50s}$ values in *Meland and Norrman*'s [1966] Figure 4.

8. We estimated $\tau^*_{c_{r50m}}$ and $Re^*_c$ values from *Shields*' [1936] Figure 6. Because the corresponding grain sizes are unreported by Shields, we assigned $D_{50m}$ values reported by each source based on a sensible match of grain size with estimated $\tau^*_{c_{r50m}}$ and $Re^*_c$ pairs. Kramer's data are included here, however, it is uncertain if they are reference- or visual-based values; Kramer measured bed load transport rates but did not report them. $D_{50m}$ and $h_c$ values for *Casey* [1935] are from *Tison* [1953].

9. Using data in *Johnson*'s [1943] Tables 27–30, we linearly extrapolated $\tau_c$ values from plots of bed load transport rate versus shear stress for each sediment mixture and used these data to calculate $\tau^*_{c_{r50m}}$ values from the Shields equation. Lack of bedload size distributions precluded analysis of nonlinear relationships between bedload transport and shear stress using a *Parker and Klingeman* [1982] type method.

10. Curiously, *Gilbert*'s [1914] data analyzed in this fashion are very different than the other reference-based data and are excluded from our analysis.

11. We applied the method of note 7 to *Meland and Norrman*'s [1969] Figure 5, with $D_{50m} \approx 3.9$ mm [*Meland and Norrman*, 1969].

12. Although *Paintal* [1971] questions the existence of a definitive threshold for mobility [see also *Lavelle and Mofjeld*,

1987], two potential $\tau^*_{c_{r50m}}$ values can be estimated from his analysis. Extrapolating high bed load transport rates to a zero value yields $\tau^*_{c_{r50m}} \approx 0.05$ for $D_{50m}$ values of 2.5 and 7.95 mm [*Paintal*, 1971, Figure 8]. The resultant $\tau^*_{c_{50}}$ and $Re^*_c$ pairs agree with other referenced-based values determined by unreported curve-fitting techniques [e.g., *Shields*, 1936]. However, a more appropriate nonlinear fit of *Paintal*'s [1971, Figure 8] data can yield $\tau^*_{c_{50}}$ values as low as 0.01, depending on the chosen reference bedload transport rate.

13. We corrected *Mizuyama*'s [1977] modified Shields equation for a neglected buoyancy term (left-hand side of his equation (3.27); see work by *Wiberg and Smith* [1987] for a similar correct derivation). Using this corrected equation, we calculated $\tau^*_{c_{r50m}}$ values with data from *Mizuyama*'s [1977] Tables 3.1 and 3.2; $\tau^*_{c_{r50m}}$ values using a traditional Shields equation are shown in parentheses for comparison. $\Phi$ values used by *Mizuyama* [1977] are mass angles of repose for the bulk sediment mixture [sensu *Miller and Byrne*, 1966], rather than intergranular values.

14. The $\tau^*_{c_{r50m}}$ is for the lowest dimensionless bed load transport rate ($10^{-6}$) of the composite data set [*Pazis and Graf*, 1977, Figure 3].

15. We calculated $\tau^*_{c_{r50m}}$ values using *Bathurst et al.*'s [1987] equation (15.1) and values in their Table 15.3; equivalent $\tau^*_{c_{r50m}}$ values determined from a traditional Shields expression are shown in parentheses for comparison. We estimated $Re^*_c$ values from *Bathurst et al.*'s [1987] Figure 15.3. As with *Mizuyama* [1977], $\Phi$ is the mass angle of repose of the sediment mixture.

16. We estimated $\tau^*_{c_{r50m}}$ values from *Bathurst et al.*'s [1987] Figure 15.3. Equivalent $\tau^*_{c_{r50m}}$ values using a traditional Shields expression are shown in parentheses [*Bathurst et al.*, 1979, Table 6].

17. We calculated $\tau^*_{c_{r50m}}$ from the Shields equation using $\tau_{c_{r50m}}$ of *Li and Komar*'s [1992] Figure 1b.

18. Using *Day*'s [1980] Figure 9 we determined $\tau^*_{c_{r50m}}$ values for dimensionless particle sizes corresponding to reported $D_{50m}$ values [*Day*, 1980, Table 1], recognizing that $\tau^*_c$ is the square of the Ackers-White mobility number.

19. We estimated $\tau^*_{c_{r50m}}$ for run 3 from *Day*'s [1981] Figure 2 using $D_{50m}$ of *Day*'s [1981] Figure 1 and recognizing that $\tau^*_c$ is the square of the Ackers-White mobility number.

20. We calculated $\tau^*_{c_{r50m}}$ from the *Parker and Klingeman* [1982] method as modified by *Ashworth and Ferguson* [1989] using data reported in *Leopold and Emmett*'s [1976, 1977] Tables 1 and 2.

21. Using the Shields equation we developed power law functions for $\tau^*_{c_{ri}}$ from data in *Wilcock*'s [1992a] Figure 6.5 and Table 6.2.

22. We used the same procedure as in note 5, but with $D_i = D_{50m}$ estimated from the bulk bed distribution of *Wilcock and McArdell*'s [1993] Figure 5.

23. *Coleman* [1967] reports critical boundary Reynolds numbers in terms of $u$ (the flow velocity measured at a height of $0.5D_{50s}$) rather than $u^*$. Consequently, his $Re_c$ values must be converted to $Re^*_c$ values by replacing $u$ with $u^*$. We used $Re^*_c$ values estimated from *Coleman*'s [1967] data by *Fenton and Abbott* [1977], but we did not use their corrected $\tau^*_{c_{50}}$ values, as *Coleman*'s [1967] shear stresses are not calculated from measures of $u$ but instead are based on direct measures of strain and can be read from his Figure 3 without need for conversion. *Fenton and Abbott*'s [1977] $\tau^*_{c_{50}}$ values are shown for comparison in parentheses.

24. Reported data were derived from *Gilbert*'s [1914] Table 10 using his definition of incipient motion (i.e., "several grains moving" from a plane-bed surface [*Gilbert*, 1914, pp. 68, 71]) which we consider similar to *Kramer*'s [1935] "medium movement."

25. We calculated the first two $\tau^*_{c_{v50m}}$ and $Re^*_c$ pairs from *White*'s [1940] Table 1 (experiments 1a, 1bii, and 2a) assuming $\rho_s = 2650$ kg/m$^3$ and $\rho \approx 900$ kg/m$^3$; $\rho$ was estimated by comparing the reported depth-slope products [*White*, 1940, Table 1, "from d"] with those calculated for a fluid medium of water. We calculated the third $\tau^*_{c_{v50m}}$ and $Re^*_c$ pair from *White*'s [1940, p. 328] data assuming $\nu = 10^{-6}$ m$^2$/s. All other reported values were taken from *White*'s [1940] Table 2.

26. Reported data are derived from *Grass*' [1970] averages of instantaneous shear stresses and are assumed equivalent to time-averaged values; instantaneous equivalents are shown in parentheses for comparison. *Grass*' [1970] direct shear stress measures implicitly account for sidewall effects.

27. We calculated $\tau^*_{c_{v50m}}$ from the Shields equation using the reported $u^*_c$ value for initiation of grain motion [*Wimbush and Lesht*, 1979, Table 1], $\rho = 1027.6$ kg/m$^3$ (sea water of 3.5% salinity and 7.3°C [*Todd*, 1964]) and $\rho_s = 2015$ kg/m$^3$. We estimated $\rho_s$ by averaging the densest possible carbonate (aragonite) with the least dense skeletal test measured by *Wimbush and Lesht* [1979].

28. We estimated $\tau^*_{c_{v50m}}$ by replacing the mean sand diameter in reported Shields stresses with $D_{50m}$ values determined from the full grain size data of *Young and Mann*'s [1985] Table 1. We calculated corresponding $Re^*_c$ values in a similar fashion.

29. The $\tau^*_{c_{q50s}}$ of Great Eggleshope Beck was calculated from the Shields equation, with $D_{50s}$ taken as the median grain size of the framework distribution [*Komar and Carling*, 1991, Table 1] and $\tau_{c_{q50s}}$ determined from *Carling*'s [1983] equation (7) using the above $D_{50s}$ value. The framework distribution is observationally similar to the censored (i.e., armored) surface layer distribution [*Carling and Reader*, 1982]; $\tau^*_{c_{q50s}}$ for Carl Beck was calculated in a similar fashion using a median framework gravel size estimated from *Carling*'s [1989] Figure 2, with the distribution truncated at 4 mm [*Carling*, 1989].

30. We developed a power law function for $\tau^*_{c_{qi}}$ using $D_i$ and $\tau^*_{c_{qi}}$ data from *Hammond et al.*'s [1984] Table 1 and $D_{50s} = 15.5$ mm estimated from the grab sample data of their Figure 3 truncated at 2 mm; the surface material was observationally devoid of sand [*Hammond et al.*, 1984].

31. Rather than using the *Andrew*'s [1983] equation, we regressed a power law function through *Andrews and Erman*'s [1986] Figure 7 data and evaluated $\tau^*_{c_{q50s}}$ with $D_i = D_{50s} = 58$ mm and $D_{50ss} = 30$ mm [*Andrews and Erman*, 1986].

32. Using bed load transport data from *Milhous* [1973], *Carling* [1983], and *Hammond et al.* [1984], *Komar* [1987a] developed empirical competence equations for each study site, expressing competence as a power law function between shear stress and the largest mobile grain size [*Komar*, 1987a, Table 1]. To facilitate convergence of different data sets, *Komar* [1987a] proposed that Shields stress be expressed as a power law function of the form $\tau^*_{c_{qi}} = \alpha(D_i/D_{50})^{\beta}$ (similar to that used by *Parker et al.* [1982] and *Andrews* [1983]), where $D_{50}$ is a generic term that *Komar* [1987a] inconsistently evaluated as either $D_{50ss}$ or the "crossover" grain size [*Komar*, 1987a, p. 205]. For *Milhous*' [1973] data, *Komar* [1987a] set $D_{50} = D_{50ss}$ and algebraically manipulated the competence equation into the desired form of $\tau^*_{c_{qi}}$ [*Komar*, 1987a, p. 207]. For the *Carling* [1983] and *Hammond et al.* [1984] data, *Komar* [1987a] chose $D_{50}$ values based on the empiricism that competence curves

cross the *Miller et al*. [1977] incipient motion curve at values of $D_i \approx D_{50}$, as demonstrated by *Day*'s [1980] data [*Komar*, 1987a, Figure 2], where $D_{50} = D_{50m}$ for *Day*'s [1980] data. However, the crossover value for *Carling*'s [1983] data (20 mm [*Komar*, 1987a, Figure 3]) is similar to the $D_{50ss}$ value for that site (27 mm [*Komar and Carling*, 1991, Table 1]); while the *Hammond et al*. [1984] $D_{50ss}$ value is not known, the crossover value (7.5 mm [*Komar*, 1987a, Table 1]) is less than $D_{50s}$ (15.5 mm, note 30) and more like the $D_{50}$ of their grab sample (10.3 mm [*Hammond et al*., 1984, Figure 3]) which is likely to be composed predominantly of subsurface material. These observations indicate that *Komar*'s [1987a] $\tau^*_{c_{qi}}$ equations for *Milhous* [1973], *Carling* [1983], and *Hammond et al*. [1984] [*Komar*, 1987a, Table 1] are functions of median grain sizes similar or equal to those of the subsurface (i.e., *Komar*'s [1987a] generic $D_{50}$ is defined as $D_{50ss}$ in the Milhous equation and is roughly equivalent to $D_{50ss}$ in the Carling and Hammond et al. equations).

*Komar* [1987a, Figure 4] fixed $\alpha$ values for the *Carling* [1983] and *Hammond et al*. [1984] $\tau^*_{c_{qi}}$ equations from crossover points with the *Miller et al*. [1977] curve, forcing $\alpha = 0.045$ and leaving $\beta$ to be back-calculated. *Komar* [1987a] found that the resultant $\tau^*_{c_{qi}}$ equation for *Carling*'s [1983] data appeared to describe *Milhous*' [1973] data quite well, so he used it to represent *Milhous*' [1973] data, abandoning the algebraically defined Milhous equation (cf. p. 207, Figure 5, and Table 1 of *Komar* [1987a]). Rejecting this unsound rationale, we have maintained the original algebraically defined Milhous equation in Table 1c (as also preferred by *Komar and Shih* [1992]).

We calculated $\tau^*_{c_{q50s}}$ values from these "derived" $\tau^*_{c_{qi}}$ equations using values of $D_i = D_{50s}$ culled from other sources (see following notes) and $D_{50}$ as defined by *Komar* [1987a, Table 1]. *Komar*'s [1987a] $\tau^*_{c_{qi}}$ equations for *Day*'s [1980] data were not used, as they represent $D_i/D_{50m} > 1$ only [*Komar*, 1987a, p. 209].

33. We calculated $\tau^*_{c_{q50s}}$ from the algebraically manipulated Milhous equation [*Komar*, 1987a, p. 207] and values of $D_{50} = D_{50ss} = 20$ mm [*Komar*, 1987a, Table 1] and $D_i = D_{50s} = 63$ mm [*Milhous*, 1973, Table 2; *Winter*, 1971]. We estimated a second set of values from a power law fit of data presented in *Komar*'s [1987a] Figure 5.

34. We calculated $\tau^*_{c_{q50s}}$ from the Carling equation [*Komar*, 1987a, Table 1] and values of $D_{50} \approx 20$ mm [*Komar*, 1987a, Table 1] and $D_i = D_{50s} = 62$ mm, the median grain size of the framework distribution [*Komar and Carling*, 1991, Table 1] (see also note 29). We estimated a second set of values from a power law fit of data presented in *Komar*'s [1987a] Figure 6.

35. We calculated $\tau^*_{c_{q50s}}$ from the *Hammond et al*. [1984] $\tau^*_{c_{qi}}$ equation [*Komar*, 1987a, Table 1] and values of $D_{50} = 7.5$ mm [*Komar*, 1987a, Table 1] and $D_i = D_{50s} = 15.5$ mm (note 30).

36. We calculated $\tau_{c_{q50s}}$ from the Fahnestock competence equation reported by *Komar* [1987b, Table 1], and used this value to determine $\tau^*_{c_{q50s}}$ from the Shields equation. For these calculations $D_{50s}$ was determined from the composite White River grain size data [*Fahnestock*, 1963, Table 2].

37. We calculated $\tau^*_{c_{q50s}}$ values from relevant equations [*Komar and Carling*, 1991, Figure 9] and grain sizes [*Milhous*, 1973, p. 16; *Komar and Carling*, 1991, Table 1]. Although the legend for *Komar and Carling*'s [1991] Figure 9 indicates that the Carling equation is expressed as a function of $D_{50ss}$, the normalizing grain size used by the authors (62 mm) is that of the framework gravel [*Komar and Carling*, 1991, p. 498], which is equivalent to the censored (i.e., armored) surface size [*Carling and Reader*, 1982].

38. Values of $\tau^*_{c_{q50s}}$ were determined from power functions for $\tau^*_{c_{qi}}$ developed from *Lepp et al*.'s [1993, Tables 2–4] data.

39. We used $D_i = D_{50ss} = 27$ mm [*Komar and Carling*, 1991, Table 3] and $D_{50} = 20$ mm (see note 32).

40. We determined $\tau^*_{c_{t50s}}$ values using the Shields equation and $\tau_{c_{t50s}}$ values presented in *White*'s [1940] Table 2. However, he erroneously adds $\tan\theta$ to $\tan\Phi$, rather than subtracting it [cf. *Wiberg and Smith*, 1987]; our reported values reflect this correction.

41. The $\tau^*_{c_{t50s}}$ values were estimated from *Ikeda*'s [1982] Figure 5 for dimensionless lift-to-drag ratios of 0 and 0.8 (shown in parentheses).

42. Reported $\tau^*_{c_{t50s}}$ values are means for velocity fluctuations of zero, one, and two standard deviations, respectively [*Naden*, 1987, Table 2].

43. $\Phi$ and grain protrusion measurements were made from flume-worked heterogeneous bed surfaces with plane-bed or low-amplitude topography. The specific $\tau^*_{c_{t50s}}$ value reported here was read from *Kirchner et al*.'s [1990] Figure 18 for $D/K_{50} = 1$ and $n = 10$.

44. $\Phi$ measurements were made from bed surfaces of a natural pool-riffle stream (Wildcat Creek), and protrusion values were derived from *Kirchner et al*.'s [1990] experiments. The specific $\tau^*_{c_{t50s}}$ value reported here was read from *Buffington et al*.'s [1992] Figure 13 for $D/K_{50} = 1$ and $n = 0.1$.

45. Reported $\tau^*_{c_{t50s}}$ values bracket the calculated threshold of "continuous" motion, defined as one or more particles moving at all times during a simulation [*Jiang and Haff*, 1993].

46. *Ling* [1995] proposes two Shields curves representing sediment motion characterized by lifting and rolling, respectively. These curves coalesce at high $Re^*_c$ values. The $\tau^*_{c_{t50s}}$ value reported here is half that of *Ling*'s [1995] Figure 3, corresponding to $k_s/D_{50s} = 1$ [*Ling*, 1995, p. 477].

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