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Geologic constraints on bedrock river incision using the stream power law

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Abstract. Denudation rate in unextended terranes is limited by the rate of bedrock channel incision, often modeled as work rate on the channel bed by water and sediment, or stream power. The latter can be generalized as KA^mS^n, where K represents the channel bed's resistance to lowering (whose variation with lithology is unknown), A is drainage area (a surrogate for discharge), S is local slope, and m and n are exponents whose values are debated. We address these uncertainties by simulating the lowering of ancient river profiles using the finite difference method. We vary m, n, and K to match the evolved profile as closely as possible to the corresponding modern river profile over a time period constrained by the age of the mapped paleoprofiles. We find at least two end-member incision laws, $KA^{0.3\text{-}0.5}S^1$ for Australian rivers with stable base levels and $K_fA^{0.1\text{-}0.2}S^n$ for rivers in Kauai subject to abrupt base level change. The long-term lowering rate on the latter expression is a function of the frequency and magnitude of knickpoint erosion, characterized by K_f. Incision patterns from Japan and California could follow either expression. If they follow the first expression with $m = 0.4$, K varies from 10^{-7}-10^{-6} $m^{0.2}$/yr for granite and metamorphic rocks to 10^{-5}-10^{-4} $m^{0.2}$/yr for volcaniclastic rocks and 10^{-4}-10^{-2} $m^{0.2}$/yr for mudstones. This potentially large variation in K with lithology could drive strong variability in the rate of long-term landscape change, including denudation rate and sediment yield.

1. Introduction

The fundamental constraint on the rate at which mean topography is denuded (except in extensional terranes) is the rate of bedrock channel incision. The pattern of river incision can influence crustal structure across orogens by changing the distribution of stress in the crust [*Hoffman and Grotzinger*, 1993; *Beaumont and Quinlan*, 1994] and at shorter timescales, the sediment yield through the influence of channel lowering rates on the gradients of adjacent hillslopes [e.g., *Arnett*, 1971]. Neither influence can be satisfyingly predicted from first principles of geomorphology because of a lack of knowledge about how to quantify both the processes that incise bedrock and the resistance of bedrock to these processes. If incision rate varies orders of magnitude with lithology, then the response time of catchments to climatic or tectonic perturbations should vary dramatically among geologic terranes. In contrast, if variation in incision rate with lithology is small, then the timescale of geomorphic response may be entirely a function of tectonism or climate.

In this paper we use dated river paleoprofiles from Australia, Kauai, California, and Japan to evaluate the form of a stream incision law

$$\frac{dz}{dt} = KA^mS^n , \qquad (1)$$

where dz/dt is incision rate, A is drainage area, S is channel slope, m and n are exponents whose values are debated, and K is the resistance to incision whose variation with lithology is undocumented. Area is a surrogate for discharge, which is not gauged for many rivers. We address three outstanding questions central to modeling bedrock river incision: (1) Can the difference between stream paleoprofiles and modern profiles be used to constrain the exponents on area (m) and slope (n)? (2) Given m and n, how might K vary as a function of lithology and region? (3) What parameters do we need to know best in order to calibrate K, and what are meaningful differences in K given these uncertainties? Our contributions are to calculate K and its uncertainty in a consistent manner, to examine whether the pattern of profile incision is consistent with a stream power law, and to use the pattern of incision to constrain the possible range of m and n values. Our results apply to channels dominated by fluvial incision and should not be applied to the steep portion of the channel network dominated by debris flows.

2. Previous Work on River Incision Laws

Early workers [*Gilbert*, 1877; *Gannet*, 1893] hypothesized that the rate of bedrock river lowering should be a function of rock resistance, river discharge, and slope. These ideas were first incorporated into a stream power law for bedrock incision by *Howard and Kerby* [1983], who argued that incision should be proportional to the shear stress exerted by the dominant discharge. By regressing erosion rate against formulations of the stream power law with varying exponents on slope and area (n and m, respectively), they calculated $m = 0.45$ and $n = 0.70$ for a channel incising badlands. *Seidl and Dietrich* [1992] used Playfair's law (i.e. tributaries enter at the elevation of the trunk stream) to constrain the ratio m/n for rivers in the Oregon Coast Range. They calculated area/slope ratios at tributary junctions where both K and dz/dt are assumed to be equal and found $m/n = 1$. *Seidl et al.* [1994] calibrated K and found $m = n = 1$ for valleys on the west side of Kauai by regressing erosion rate (calculated as the depth of the modern river below the

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original shield volcano surface) against (1). Importantly, they included the effect of changing boundary conditions in their values of K, m, and n, in effect lumping together points that have already responded to a base level drop with those that have yet to feel this signal. *Rosenbloom and Anderson* [1994] used m and n values from *Seidl et al.* [1994] along with a formulation of the stream power law where discharge is explicitly calculated to calibrate K in incised marine terraces of the Santa Cruz Mountains, California. These studies have not calculated K using a consistent method, so the variation of K with lithology is unknown. There is also disagreement on the magnitude of area and slope exponents and the ratio of the two [e.g., *Howard and Kerby*, 1983; *Seidl and Dietrich*, 1993; Seidl *et al.*, 1994].

3. Field Constraints and Modeling Approach

We use (1) to model the evolution of ancient river profiles preserved as strath terraces, lava/lahar flow tops, and intracanyon lava flows. More elaborate models of bedrock river incision that include the mechanics of abrasion [*Foley*, 1980a, b] or the influence of sediment supply [*Sklar*, 1996] require data that is not preserved over geologic time. We seek the simplest general formulation of (1) that will allow us to capture profile evolution without depending on parameters we cannot specify with precision. To this end, we do not account for possible downstream variations in rock mass quality or lithology that are bound to exist at some level along paleoprofiles. These are collapsed into a single parameter K. Nor do we attempt to model the evolution of boundary conditions on our profiles, because we cannot specify these with precision. Rather, we either use profiles that have gradual base level changes or we use subsections of profiles that satisfy this condition. We do not subsume variations in incision rate due to changes in the boundary condition into m and n.

In sections 3.1-3.4, we describe field sites in Australia, Kauai, Japan, and California that contain dated, former river profiles that have been abandoned as the river incised. With the exception of the California sites, we have extracted the river profiles from published papers. These field sites represent river incision through lithologies ranging from mudstones to crystalline basement over time periods from several thousand years to 20 million years. We used dated profiles from each of these regions as a geologic calibration of the stream power expression for bedrock river incision.

3.1. Australia

In Australia we use Miocene river profiles reported by *Young and McDougall* [1993] and *Bishop et al.* [1985]. Following burial by a maximum of 180 m of basalt in the Miocene, rivers in several areas of central New South Wales incised through the basalt flows and subsequently the crystalline basement of the Paleozoic Lachlan fold belt. Ancient river profiles are preserved as the contact between alluvium and the base of overlying intracanyon basalt flows. *Young and McDougall* [1993] used the continuity of these profiles to argue against post-Miocene deformation by faulting or tilting. The rivers drain west to the inland Murray basin, so their base level should not vary with Neogene sea level change. Differential rock uplift due to isostasy is minimal because mass removal has been insignificant enough over 20 Ma that flows are preserved over much of the landscape. *Lambeck and Stephenson* [1986] have calculated maximum isostatic rock uplift rates of 5 m/Ma. We simulate the evolution of four channels: the Tumbarumba (cut through medium-grained biotite granites and granodiorites), the Tumut (cut through alternating sequences of acid volcanics and conglomerates, sandstones, and limestones), the Wheeo, and the Lachlan (both cut through alternating Ordovician slate and mid-Paleozoic gneissic granitoids). The starting paleoprofile for each of these channels is the bottom of the lava flow, an underestimate of the actual incision by the thickness of the flow. Paleoprofiles are reported as intermittent lines by *Young and McDougall* [1993] for the Tumut and Tumbarumba and as points by *Bishop et al.* [1985] for Wheeo Creek (21 elevation points) and Lachlan River (7 elevation points).

3.2. Kauai

We use late Miocene profiles reported by *Seidl and Dietrich* [1994] from Kauai, the westernmost and oldest of the major Hawaiian shield volcanoes. Thin basalt flows of the Napali Formation have K-Ar ages of ~ 5.1 Ma [*McDougall*, 1964; *Langenheim and Clague*, 1987] and are preserved as interfluve tops between deeply incised channels. Modern channel profiles are of two types: straight profiles where incision from the interfluve increases marginally with distance downstream, and those with large slope changes and a downstream increase in incision associated with knickpoints. The former debouch onto the Mana Plain, an exposed coral reef insulating these channels from Pleistocene base level change as sea level fluctuated. Although they likely experienced prior base level changes, any associated knickpoints have now migrated through these linear channels. Modern profiles with prominent slope changes, or knickpoints, empty along the Napali Coast, a steep, cliff-dominated coastline arguably initiated by major landslides [*Moore et al.*, 1989]. *Seidl et al.* [1994] attribute knickpoint formation on these channels either to rapid cliff retreat following a massive landslide or to sea level change. We cannot evaluate the stream power law on the channels with current knickpoints without precisely simulating the timing and magnitude of the change in downstream boundary conditions as the paleoprofile evolves. On the other hand, once the knickpoints have migrated through the profile, the pattern of incision can be used to infer the form of the stream power law without exactly specifying the boundary conditions. Therefore we use four paleoprofiles without large knickpoints in the modern profile to investigate the incision law.

3.3. Japan

We use dated strath terraces reported by *Suzuki* [1982] from the Iwaki River, which drains the south side of the Quaternary Iwaki stratovolcano on northeast Honshu, Japan. The Iwaki and a tributary, the Sakuzawa, flow at nearly right angles to the strike of a homocline of upper Pliocene mudstones and intercalated Miocene tuffs and lava flows. Base level for the Iwaki is the sub-100-m Tsugaru plain, composed of alluvial fans grading downward into marsh and deltaic plains [*Umitsu*, 1976].

3.4. California

We report river profiles from California (Figure 1) that are incised into lahars of the Pliocene Tuscan Formation [*Helley and Harwood*, 1985] on the east side of the Great Valley (specifically, the Oak Creek, Bella Vista, and Palo Cedro U.S. Geological Survey 7.5' quadrangles). Flows are locally covered by pediment gravels whose age is elsewhere bracketed by basalt and tephra between 0.45 and 1.08 Ma [*Helley and Jaworowski*, 1985]. The pediment is interpreted to record blockage of throughgoing valley drainage, with base level dropping below the slope break at ~225 m elevation (Figure 1) between 1.08 and 0.45 Ma. The paleoprofiles in Figure 1 are projections of the flat interfluves onto the modern river profile every 1.5-6.1 m, depending on the map con-

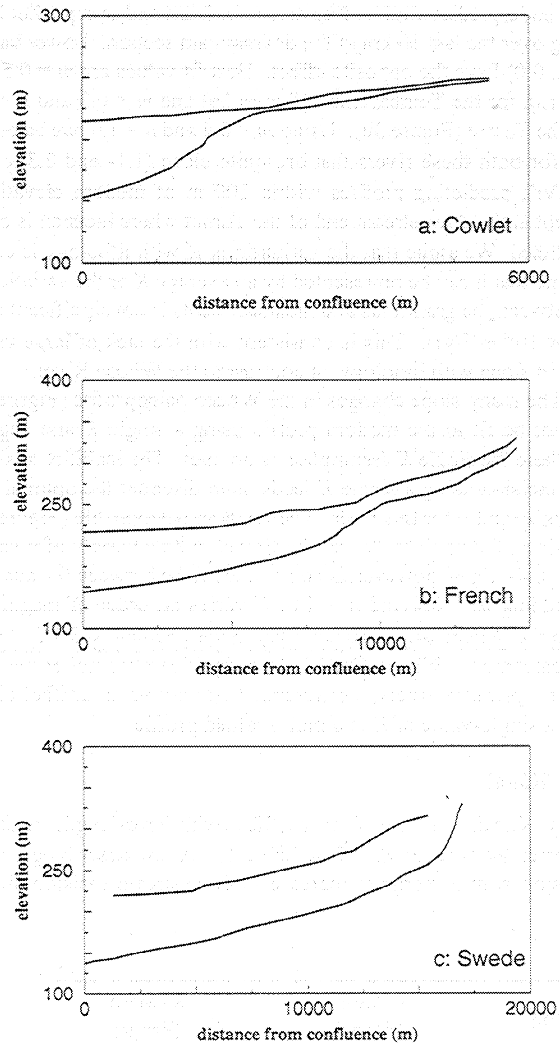

Figure 1. Modern and ancient profiles from adjacent channels, northeast Great Valley, California. The knickpoint still apparent on the modern (a) Cowlet and (b) French profiles has already migrated up (c) Swede Creek. The bench on the paleoprofile (just above 200 m elevation) is the likely former base level for these channels.

tour interval. Knickpoint propagation appears to depend on basin size, as the smaller basins have knickpoints that the largest (Swede Creek) lacks. To avoid having to simulate a boundary condition with an unknown lowering rate at the bottom of Cowlet and French Creeks, we truncate their paleoprofiles above the large knickpoint and use the upstream segments to estimate m, n, and K. We discuss this issue further in section 5.2.

3.5. Morphometric Parameters

For each of the field site catchments, we parameterize drainage area as a function of downstream distance by tracing drainage area and stream length from topographic maps. We then fit drainage area (A, in km^2) as a power function of down-channel distance (x) from the start of the trunk stream (blue lines on topographic maps). With $A = Dx^e$, where D and e are fitted parameters, (1) becomes

$$\frac{dz}{dt} = K\left(Dx^e\right)^m S^n . \tag{2}$$

Given a topographic map from which one can calculate D and e and a dated paleoprofile as an initial condition, there are three unknowns: K, m, and n.

3.6. Description of m

The exponent m represents a discharge-drainage area relation that is weighted by the importance of discharge on incision and could vary between river basins independently of annual rainfall because of variations in hypsometry or hydrologic processes. If this interbasin variation is important, no single value of m describes incision in the stream power law (1). For bankfull discharge (Q_b), however, the range $Q_b \propto A^{0.65-0.80}$ is thought to be robust for many basins [*Leopold et al.*, 1964]. We do not account for variations in unit stream power (i.e., discharge per unit channel width) because we do not know it for the study sites and cannot retrodict it with precision. Hence m also contains the relation between discharge and channel width (w), commonly $w \propto Q_b^{0.5}$ for bankfull discharge on alluvial channels [*Leopold et al.*, 1964]. Although it is formally incorrect to manipulate the preceding regression relations algebraically, we can estimate a range for the value of m by combining them:

$$\frac{Q_b}{w} = \frac{Q_b}{Q_b^{\ 0.5}} = Q_b^{\ 0.5} = \left(A^{0.65-0.80}\right)^{0.5} = A^{0.3-0.4} .$$

For a channel whose width increases as the square root of bankfull discharge and in which bedrock incision occurs predominately during bankfull or larger discharges, we expect that $0.3 < m < 0.4$.

3.7. Description of Numerical Routine

Profile points are interpolated to a uniform spacing between original data points using a second-order polynomial spline. These points are evolved with a finite difference code that integrates (1) using a fifth-order Runge-Kutta method [*Press et al.*, 1992], a version of Euler's method. Slope at each point is calculated using the two nearest neighbors. Slope at the upstream boundary is extrapolated from the three nearest points at each iteration. Slope at the downstream boundary is extrapolated from the three nearest upstream points of the initial paleoprofile and held constant. Hence the downstream boundary condition is one of constant lowering at a rate determined by the initial slope, m, n, and K. We assume that parameter and variable errors are normally distributed so that the maximum likelihood estimator of a good model fit to the modern river profile is the minimized sum of the squares (ssq) of the differences between model and modern elevation, hereafter, the mismatch.

We use a brute force two-parameter search to estimate the values of m and n from paleoprofiles, varying m and n by increments of 0.1 between 0.1 and 2.0, a range within which we find well-defined minima for the mismatch. K is then optimized for each combination using a procedure described in the next paragraph. For example, for each guess of n from 0.1 to 2.0, 20 curves are generated (one for each m guess), each plotting the mismatch as a function of optimized K for a particular m. The curve with the lowest mismatch represents the optimized m for the choice of n. From the collection of 400 curves for m and n for each river, we choose the combinations that result in the lowest mismatch (Table 1).

We then investigate the variation of K with region and lithology using our estimates for m and n. We vary K using a golden section

Table 1. Summary of Optimized m and n

Region	River	m	n	m/n
Kauai	Kaulaula	0.1 - 0.2	nc	nc
	Waiaka	0.1 - 0.2	nc	nc
	Waipao	0.1 - 0.2	nc	nc
	Paua	0.1 - 0.2	nc	nc
Australia	Tumbarumba	0.5	1	0.5
	Tumut	0.3	1	0.3
	Wheeo	0.1	0.5 - 0.6	0.2
	Lachlan	0.5	0.5	1
California	Cowlet	0.1 - 0.2	1.6 - 1.7	0.1
	French	0.2	0.1	2
	Swede	nc	nc	n - 0.1
Japan	Iwaki	>2	0.1	nc
	Sakuzawa	nc	nc	n - 0.1

Here nc = not constrained.

search (a variation on the bisection method of finding roots) until we minimize the sum of the squares measured between (1) the model output integrated over the age of the paleoprofile, and (2) the modern river profile. The K that minimizes this mismatch for a profile is referred to as the optimal K and is shown in Table 2 for two end-member river incision laws represented by Australian profiles (K) and Kauai profiles (K_f).

4. Results

4.1. Australia

As *Young and McDougall* [1993] assert, the pattern of incision on the Tumut and Tumbarumba Rivers is described by a stream power law (Figures 2 and 3). We find that higher values of m (e.g., 1.0) underpredict observed upstream incision and overpredict lowering over the last 10 km of the downstream section. Lower values (e.g., 0.0) have the opposite effect. Best fit values are $m = 0.5$ and $n = 1.0$, for the Tumbarumba (Figure 2b) and $m = 0.3$ and $n = 1.0$ for the Tumut (Figure 3b). Using $m = 0.4$ and $n = 1.0$, we calculate K's for both these rivers that are quite close (1.1- and 2.3 x 10^{-6} $m^{0.2}$/yr), predicting profiles within 100 m of modern elevations, except at the downstream end of the Tumut where incision is overpredicted. We argue that the variation in K with lithology is either so fine that it can be represented by an average K or the variation in K between the granitoids and metasediments is not significant (i.e., $\Delta K < 10^{-6}$ $m^{0.2}$/yr). This is consistent with the lack of large variation in slope with lithology, in contrast to the Wheeo River.

The many slope changes in the Wheeo paleoprofile (Figure 4a) cannot be fit to the modern profile using a single K and suggest that here the single K assumption is not met. The inability to simulate incision using a single K leads us to discount the optimal values of m and n for this river. The Lachlan paleoprofile (Figure 4b) consists of six points that can be fit well with a variety of n values from 0.4-1.0; m, however, is constrained to be between 0.4 and 0.5. Assuming $m = 0.4$ and $n = 1.0$, K varies an order of magnitude from the Wheeo (4.4 x 10^{-7} $m^{0.2}$/yr) to the Lachlan (4.3 x 10^{-6} $m^{0.2}$/yr) catchment. Wheeo Creek's K is significantly smaller than the other Australian rivers, a difference that may be an artifact of fitting a single-value of K to a multi-valued profile.

4.2. Kauai

In Kauai, we use four profiles with area-length relations reported by *Seidl et al.* [1994, Table 1]. A law describing profile incision with a marginal increase in downstream incision should

Table 2. Summary of Optimized K

Region	River	Area = Dx^c	Paleoprofile Age and Range, Myr[a]	Lithology	K, Mean (Range)	K_f, Mean (Range)
Kauai[b]	Kaulaula	$116.5x^{1.2}$	5.1 (5.0 - 5.2)	basalt		6.7 x 10^{-6} (5.2 - 8.4 x 10^{-6})
	Waiaka	$36.2x^{1.3}$	5.1 (5.0 - 5.2)	flows		6.2 x 10^{-6} (4.5 - 8.0 x 10^{-6})
	Waipao	$12.7x^{1.4}$	5.1 (5.0 - 5.2)	basalt		7.3 x 10^{-6} (5.6 - 9.1 x 10^{-6})
	Paua	$190.8x^{1.1}$	5.1 (5.0 - 5.2)	flows		3.8 x 10^{-6} (2.9 - 4.7 x 10^{-6})
Australia	Tumbarumba	$0.66x^{1.54}$	21.7 (21.3 - 22.1)	granitoids and	1.1 x 10^{-6} (1.0 - 1.3 x 10^{-6})	
	Tumut	$0.81x^{1.60}$	21.7 (21.3 - 22.1)	metasediments	2.3 x 10^{-6} (2.1 - 2.4 x 10^{-6})	
	Wheeo	$0.50x^{1.70}$	19.4 (19.2 - 19.6)	granitoids and	4.4 x 10^{-7} (4.1 - 4.8 x 10^{-7})	
	Lachlan	$4.2x^{1.33}$	19.7 (18.3 - 21.1)	metasediments	4.3 x 10^{-6} (3.1 - 5.6 x 10^{-6})	
California	Cowlet	$0.64x^{0.76}$	0.9 (0.45 - 1.3)	volcaniclastics	8.2 x 10^{-5} (4.6 x 10^{-5} - 2.0 x 10^{-4})	8.5 x 10^{-6} (4.6 x 10^{-6} - 2.1 x 10^{-5})
	French	$1.3x^{1.13}$	0.9 (0.45 - 1.3)	volcaniclastics	4.8 x 10^{-5} (1.5 x 10^{-5} - 1.5 x 10^{-4})	2.2 x 10^{-5} (7.6 x 10^{-6} - 6.6 x 10^{-5})
	Swede	$2.1x^{0.93}$	0.9 (0.45 - 1.3)	volcaniclastics	3.0 x 10^{-4} (1.7 - 4.0 x 10^{-4})	7.6 x 10^{-5} (4.4 x 10^{-5} - 1.8 x 10^{-4})
Japan	Iwaki	$12x^{0.93}$	0.005 (0.004-0.006)	mudstones, and	7.0 x 10^{-3} (4.5 x 10^{-3} - 1.1 x 10^{-2})	3.8 x 10^{-3} (2.5 - 5.8 x 10^{-3})
	Sakuzawa	$0.78x^{1.47}$	0.005 (0.004-0.006)	volcaniclastics	4.7 x 10^{-4} (2.1 - 8.5 x 10^{-4})	1.3 x 10^{-3} (5.9 x 10^{-4} - 2.3 x 10^{-3})

[a]Ages are from references cited in text.
[b]Length-area relations from *Seidl et al.* [1994].

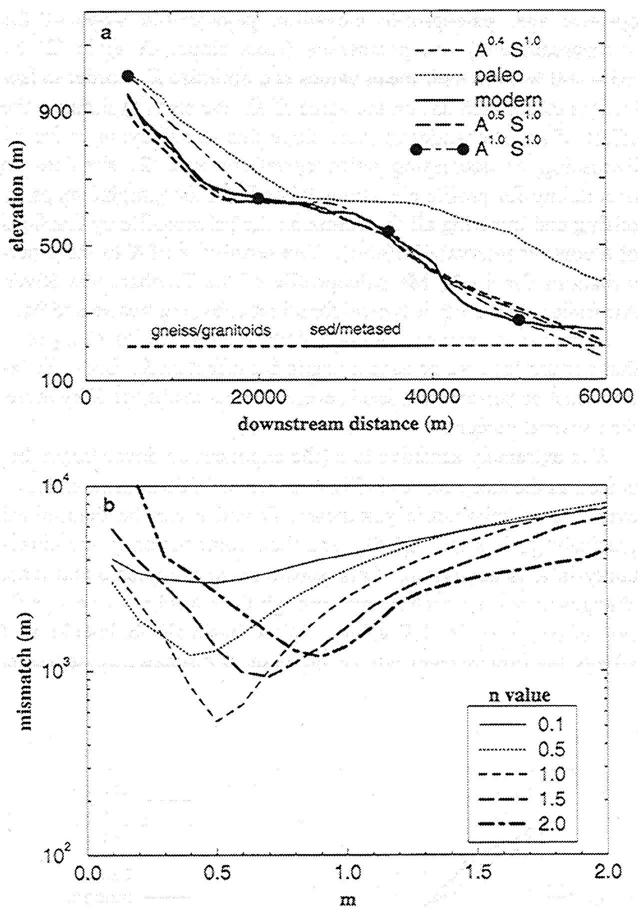

Figure 2. (a) Channel profiles for the Tumbarumba River, Australia, including the best fit stream power model with $m = 0.5$ and $n = 1.0$ (partially obscured by the modern profile in the upstream reaches). Note the poor fit upstream for the model with $m = n = 1.0$. (b) Subset of multiparameter search results for the Tumbarumba, indicating optimal values of $m = 0.5$ and $n = 1.0$.

not have a significant exponent on area, and the dramatic difference between the best fitting model profiles using $m = 1.0$ and those with $m = 0.1$ and 0.0 supports a small to zero m value (Figure 5a). All four profiles have best fit m values between 0.1 and 0.2 (see Figure 5b). Incision of these profiles is largely independent of drainage area over the long term. Optimal n values vary from 0.1 to 0.3 (Paua, Waiaka, Waipao) and 1.8 to 2.0 (Kaulaula), although a value of less than 10 m separates these fits from those of other possible n values. Thus the small range in slope along these profiles (average slopes between lava flows vary from 0.08 to 0.09) permits a wide range of n values (Figure 6). The large values for Kaulaula are an artifact of an abrupt increase in incision associated with a knickpoint at its lower end (Figure 6). K_f values for Kauai are internally consistent (Table 2).

4.3. Japan

In Japan, we examine a 5 Ka fluvial terrace on both the Iwaki and Sakuzawa Rivers. The modern Iwaki River (Figure 7a) cannot be fit well with any combination of m and n between 0.0 and 2.0 because of the large slope change in the paleoprofile. *Suzuki* [1982] has suggested that this 30-m-high knickpoint is consistent with tilting subsequent to abandonment, and consequently, the poor fit and optimal m and n values are probably a function of not including this change in boundary condition. The Sakuzawa model (Figure 7b) fits its modern profile equally well for values of $m = (n - 0.1)$ from 0.2 to 2.0, except for its lowermost section, where a knickpoint is likely propagating upward. Given the uncertainty in the appropriate m and n values, we use m and n values from both previous end-member styles of incision (seen in Australia and Kauai) to calculate both possible K values (see Table 2). For $m = 0.4$ and $n = 1.0$, K values from these profiles incised into mudstones and volcaniclastic rocks are from 1 to 5 orders of magnitude larger than those from Californian or Australian profiles with qualitatively harder rocks. Alternately, with $m = 0.1$ and $n = 0.2$, K_f values are 2 to 3 orders of magnitude larger than those from Kauai or California.

4.4. California

Paleoprofiles from both French and Cowlet Creeks have optimal m values between 0.1 and 0.2, while optimal n values range from 1.6-1.7 (Cowlet) to 0.1 (French). Swede Creek, which has already had a slope change due to base level lowering propagate along its entire length, has $m = (n - 0.1)$ and does not constrain m and n values further (analogous to the Sakuzawa profile). The inconsistency of best fit parameter values between channels and the unusually small differences in the mismatch for variations of m and n (mismatch differences are <2 m) lead us to compare model profiles for all three channels using both the Australian and Kauai

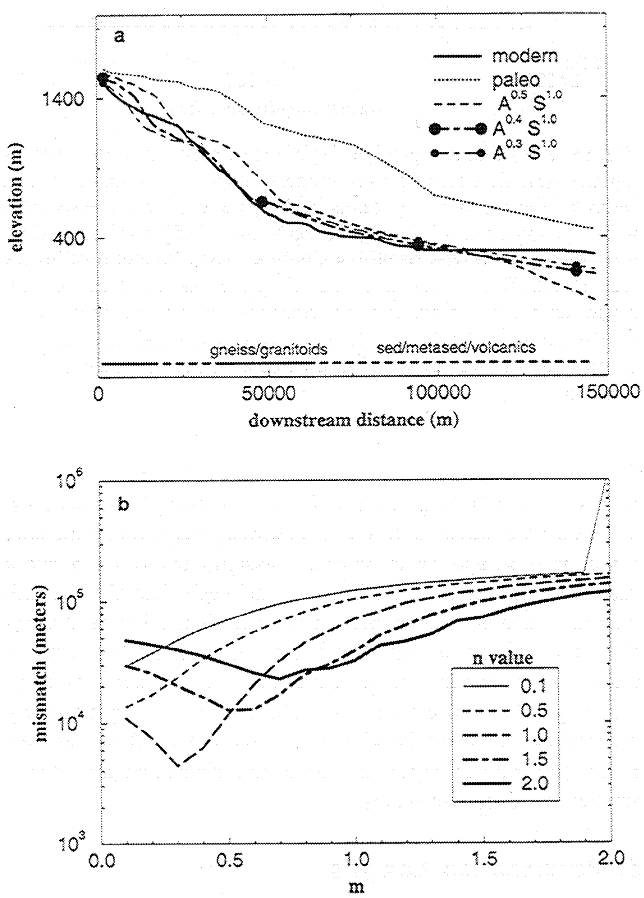

Figure 3. (a) Channel profiles for the Tumut River, Australia, including the best fit stream power model with $m = 0.3$ and $n = 1.0$. (b) Subset of multiparameter search results for the Tumut, indicating optimal values of $m = 0.3$ and $n = 1.0$.

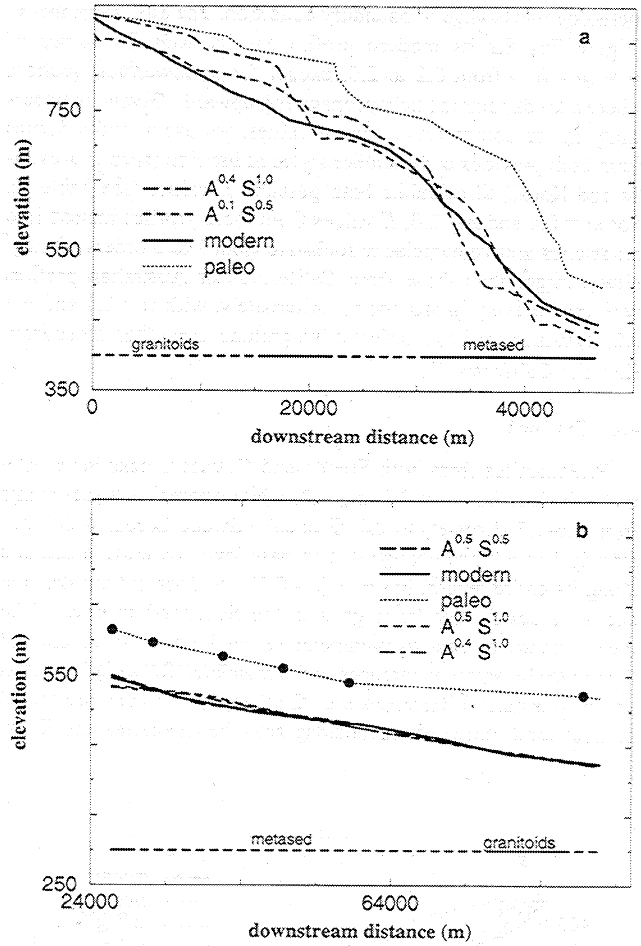

Figure 4. (a) Channel profiles for Wheeo Creek, Australia, including the best fit stream power model with m = 0.1 and n = 0.5. Many of the sharp slope breaks in the paleoprofile are associated with lithologic variation [see *Bishop et al.*, 1985], and the modern profile cannot be well fit with a single K. (b) Channel profiles for the Lachlan River, Australia, including the best-fit stream power model with m = 0.5 and n = 0.5 (obscured by modern profile). A sparse number of paleoprofile elevations (shown as solid circles) contribute to a wide range of acceptable n values for this pattern of incision.

results for m and n (Figure 8). With the exception of Swede Creek, there are no systematic differences between the two end-member model profiles, and we are unable to assign unequivocal m and n values for these channels. Hence we again calculate K using end-member m and n values from Australia and Kauai. For the Australian values of m = 0.4 and n = 1.0, K values from these profiles incised into volcaniclastic rocks are from 1 to 3 orders of magnitude larger than those from Australian profiles with qualitatively harder rocks. Alternately, with m = 0.1 and n = 0.2, K_f values are 1 order of magnitude larger than those from Kauai and consistently smaller than Japanese values.

5. Perturbation Analysis

5.1. Stream Power Law

To recognize the variation in K due solely to errors in input data, we calculated the precision of K as a function of uncertainties in field or lab data. We sequentially perturb variables (paleoprofile age, paleoprofile elevation, paleoprofile slope at the downstream end) and parameters (local slope, D, e) in (2) by $\pm$5 – 100 % from their mean values and optimize K in order to isolate the effect each has on the value of K. We also (1) simulate the effect of small changes in local slope due to surveying errors by increasing or decreasing point elevations and (2) simulate an uncertainty for profile elevations taken from topographic maps by raising and lowering all the points on the paleoprofile by fractions of a contour interval (5-100%). The sensitivity of K to these perturbations for a ~21 Ma paleoprofile of the Tumbarumba River, Australia (Figure 9a), is typical for all profiles (Figures 9b and 9c).

Except at the downstream boundary, small elevation changes that perturb local slope have a negligible effect on K. Local elevation must be perturbed at least several meters to change K by more than several percent.

K is extremely sensitive to e (the exponent on downstream distance), as the steep curves in Figures 9a and 9b demonstrate. However, the morphometric parameters D and e can be determined precisely (to less than $\pm$2 %), and their contribution to the uncertainty in K is negligible. This sensitivity does indicate that large changes in basin morphometry through time would have a significant effect on optimal K values. Initial basin shape, insofar as it affects the downstream rate of increase in discharge or sediment

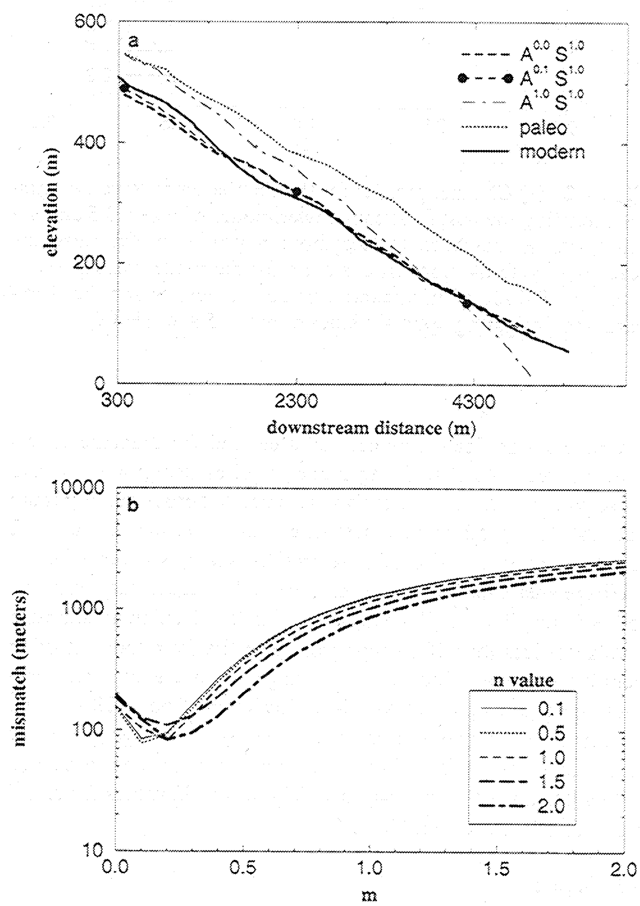

Figure 5. (a) Channel profiles for the Waiaka River, Kauai, including the best fit stream power model with m = 0.1 and n = 0.2. This model profile is not significantly different from one in which m = 0.0 and n = 1.0, but when m = n = 1.0, the model profile fits very poorly. (b) Subset of multiparameter search results for the Waiaka, indicating optimal values of m between 0.0 and 0.2. Slope varies little along these Kauai profiles, and n is poorly constrained.

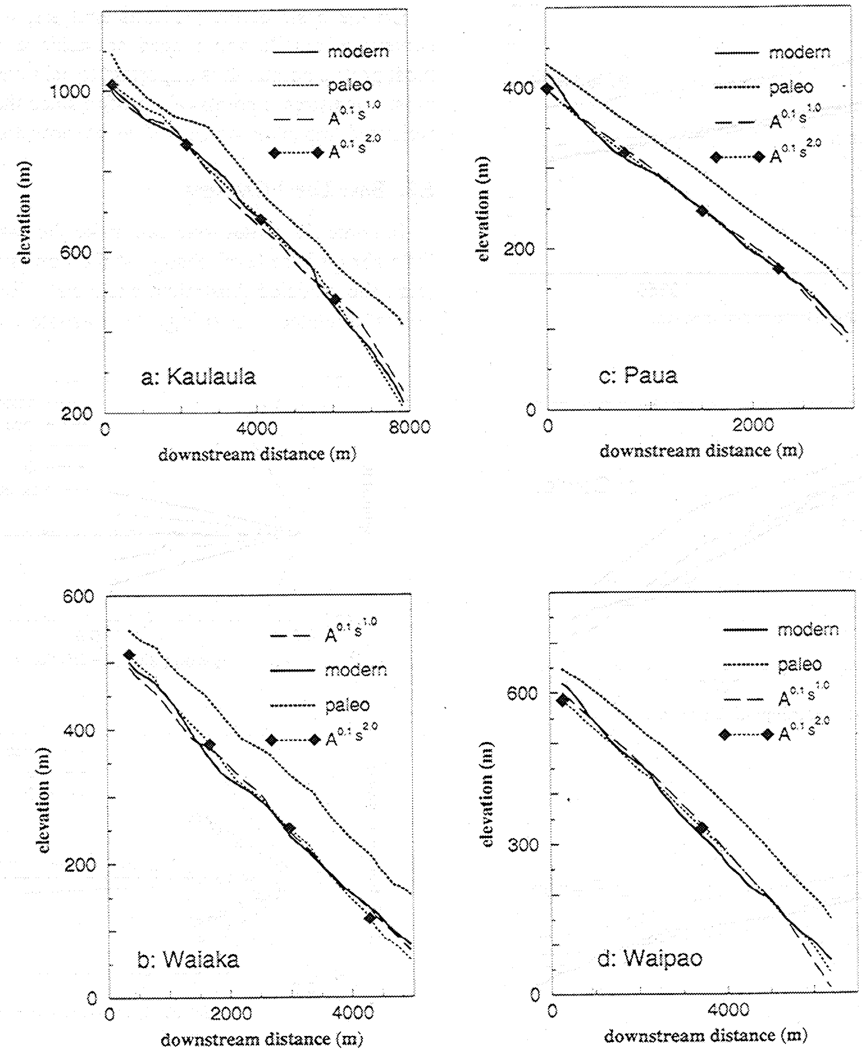

Figure 6. Profiles for Kauai channels showing stream power models with n = 1.0 and 2.0. Results illustrate the difficulty of constraining n, as small differences between models and modern profiles may also be due to variation in K along the profile.

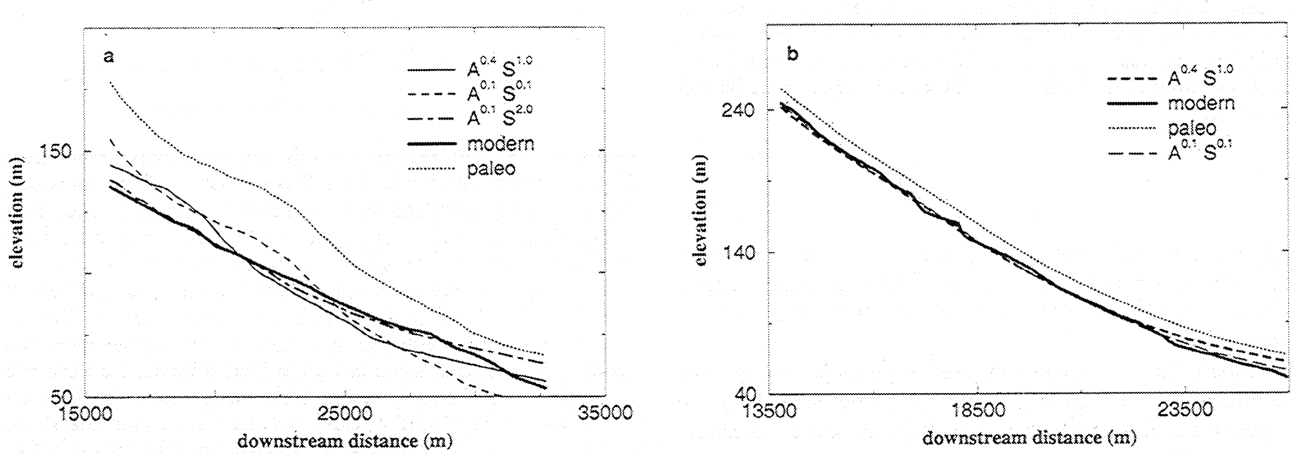

Figure 7. (a) Channel profile for the Iwaki River, Japan, including the best fit stream power model with m = 0.1 and n = 2.0. The large slope change in the paleoprofile may be the result of localized rock uplift, invalidating the optimal m and n values obtained. (b) Channel profiles for the Sakuzawa River, Japan. The pattern of incision here can be fit equally well for any value of m = (n-0.1).

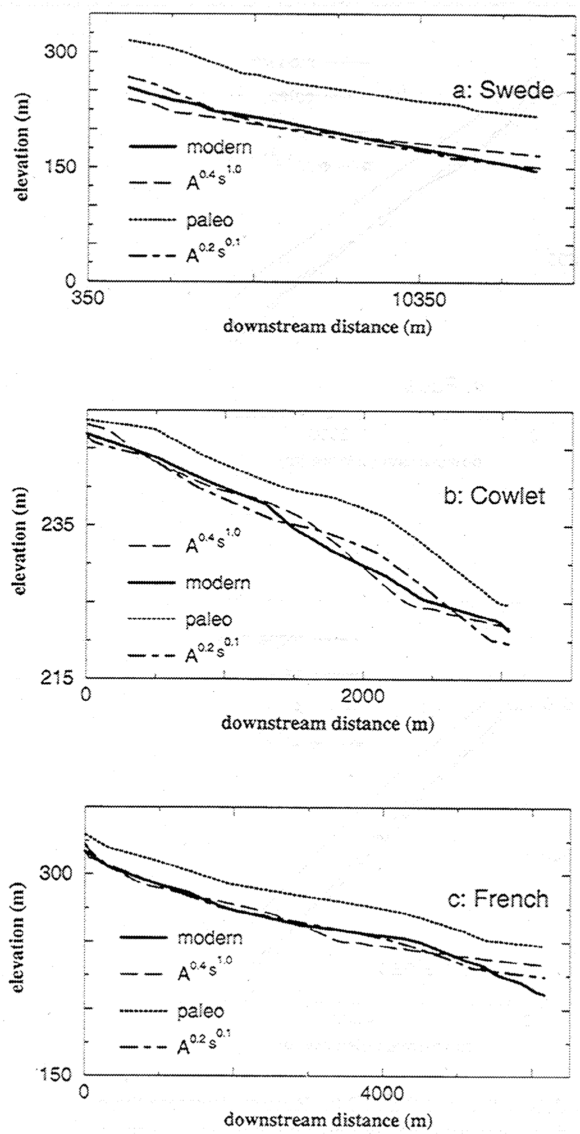

Figure 8. (a) Channel profile for Swede Creek, California, where the pattern of incision can be fit equally well for any value of $m = (n\text{-}0.1)$. Channel profiles for the truncated (b) Cowlet and (c) French Creeks, including end-member stream power models from the Kauai and Australian examples. The difference between the two end-member stream power models is not systematic enough to discriminate between them, as local differences in fit could be due here to variation in K along the profile or non-steady state boundary conditions.

supply along a river's profile, may have as fundamental a role in profile shape as rock uplift patterns or lithology (e.g., elongate basins with small e values should have profiles tending to be less concave).

Uncertainties in paleoprofile age (typically >10%) and paleoprofile elevation (usually at least 10 m) are large enough to dominate the uncertainty in estimating K if basin shape is precisely determined. Hence we combine potential errors in age (t) and profile elevation (z) to calculate maximum (t_{min} and z_{max}) and minimum (t_{max} and z_{min}) possible K values (Table 2). With this approach, we find that variations in K less than ~100% of its value could be due solely to input uncertainty.

On the basis of the previous analysis, we conclude that field surveys of profile points need not achieve submeter precision at most profile points. It is important to achieve high accuracy on the most downstream points of a profile since that slope has a heavily weighted impact on optimal K, as we now discuss.

5.2. Base Level Changes

In some field sites, one can make the case that there has been little abrupt base level change at the downstream end of a profile (e.g., the modeled Australian channels). Elsewhere, large knickpoints unrelated to lithology demonstrate that base level changes

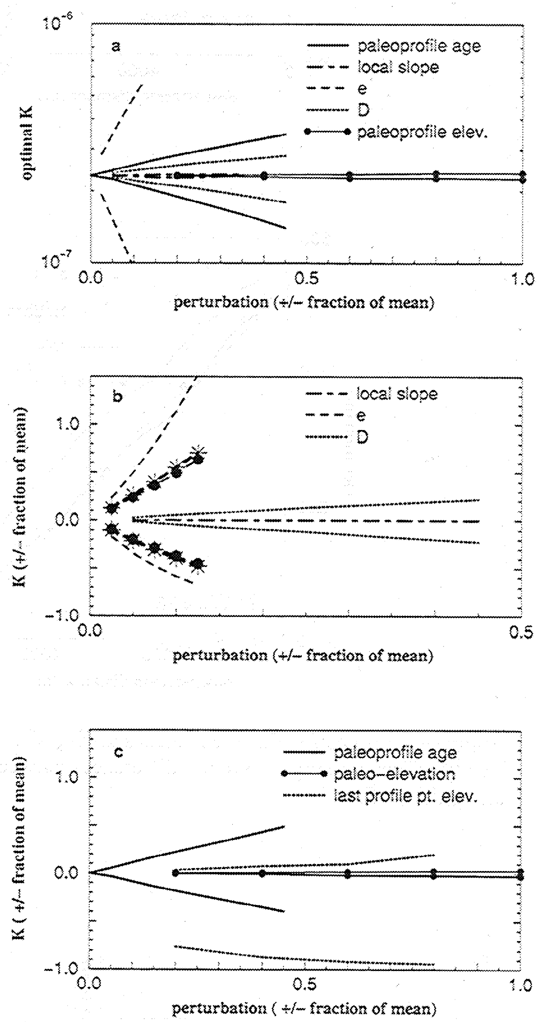

Figure 9. (a) Perturbation analysis for the Tumbarumba River showing the change in calculated K as a function of fractional perturbations away from the mean values of paleoprofile age, local profile slope (evaluated using the two nearest neighbors), paleoprofile elevation above the modern river, and D and e, constants from the relation between drainage area and distance downstream ($A = Dx^e$). (b) The analysis for fractional perturbations in K away from its mean value with fractional perturbations of the mean values of e, D, and local slope using the Tumbarumba, the Paua and the Iwaki Rivers for comparison. Note that symbols for the additional Iwaki River (solid circles) and the Paua River (stars) are shown only for the parameter e because perturbations of other parameters yield a very similar response for all three rivers. (c) The analysis for fractional perturbations in K away from its mean value due to fractional perturbations of the mean values of paleoprofile age, elevation above the modern river and elevation of the most downstream paleoprofile point.

have occurred (e.g., California, Kauai). To estimate the magnitude of this effect on K, we simulate instantaneous base level changes at model time zero by lowering the most downstream point on the paleoprofile a fraction of the distance to the modern river profile (Figure 9c). We impose the base level change at model time zero in order to determine the largest possible effect of base level lowering. Optimal K is not sensitive to raising the farthest downstream paleoprofile point toward the elevation of its nearest upstream neighbor (the low slopes this creates take nearly the same K to find a best fit). It is very sensitive to lowering the farthest downstream point, as this creates a steep slope that propagates upstream, reducing the estimated K by an order of magnitude and increasing the minimum possible misfit.

If knickpoints presently are migrating through the modeled channel but are absent from the paleoprofile and are not specified as a boundary condition, the optimum ssq does not yield an accurate value of K. We do not know the precise elevation history at the bottom end of the California and Kauai profiles, so we do not specify the appropriate base level boundary conditions. Instead, we use subsections of the paleoprofiles above modern knickpoints to calibrate K in the Californian profiles. By truncating these paleoprofiles, we lose a fraction of the lowering rate of the last point by removing it from contact with downstream points whose larger drainage areas require more rapid lowering. These optimized K values then reflect any unpreserved knickpoints that have propagated past the downstream truncation point as well as some uncertainty in the lowering rate of the last point.

6. Discussion

The incision patterns of the Australian Tumut and Tumbarumba Rivers are best fit with a stream power law where the exponent on area varies from 0.3 to 0.5, and the exponent on slope is 1.0. Despite mapped variation in lithology, both of these profiles can be reasonably well fit with a single K. The Lachlan River is also best fit by an area exponent between 0.4 and 0.5, although the sparse number of paleoprofile points allow a range of n values. By contrast, Wheeo Creek contains abrupt slope changes associated with lithology and cannot be fit well with a single K. Hence best fit exponents for m and n from this profile are not reliable. Values of m between 0.3 and 0.5 are consistent with basic hydraulic relations between channel width and bankfull discharge outlined previously, although these hydraulic relations are established for alluvial rather than bedrock rivers. This result implies that on rivers with relatively stable base levels, incision occurs along the entire profile concurrently, perhaps during extreme events (e.g., at $Q \propto A^{0.65-0.80}$) when the active bedload layer reaches bedrock. In this type of system, K would include both the frequency of discharges capable of eroding bedrock as well as the resistance of the bedrock to erosion. Width and climate-sensitive drainage area-discharge relations may uniquely specify the value of m, but we cannot at present test this hypothesis.

In contrast to the Australian profiles, the Kauai examples have a marginal area dependence on incision where $0.1 < m < 0.2$. However, the low variation in the slope of the paleoprofile does not permit a unique determination of n. The observed difference in area dependence is associated with channels where upstream propagation of large changes in slope, or knickpoints, are the likely requisite for bedrock lowering [*Seidl et al.*, 1994]. *Seidl and Dietrich* [1992] coined the term knickpoint processes to refer to incision associated with such knickpoints. We suggest that on rivers where knickpoint-driven changes in slope expose the bedrock more frequently than floods, the long-term rate and pattern of incision are set by the amplitude and frequency of such knickpoints. In these rivers, bedrock incision occurs at comparatively low rates, if at all, in the alluviated reaches outside the steepened slope of the knickpoint reach and proceeds according to the stream power law in the knickpoint reach. The long-term pattern of incision thus depends on how the amplitude of the knickpoint changes with distance upstream, rather than the stream power law (1). This would result in a pattern of incision with a low area dependence if knickpoint heights decay upstream or terminate at low drainage areas with steep slopes. Although long-term incision as a result of this process might be approximated by a stream power law with a low drainage area dependence, this scenario implies that K_f represents boundary conditions through the frequency and magnitude of knickpoints, rather than just the resistance of the rock to incision.

Our calculated K_f values for Kauai are larger than those of *Seidl et al.* [1994] because of our choice of $m = 0.1$ on the basis of multiparameter analysis. We have replicated K values reported by *Seidl et al.* [1994], using their choice of exponents, and find little difference in the magnitude of the resulting constant between our numerical integration method and their regression method. We suspect, however, that their values of $m = n = 1$ result from calibration using rivers with downstream increase in cumulative erosion due to knickpoints from changes in base level, rather than large values of m. We suggest that until boundary conditions have propagated through the modern profile, fitting a largely linear paleoprofile to a modern profile with knickpoints results in an implicit inclusion of boundary conditions into m and n. This is undesirable because it makes these exponents a strong function of the history of local boundary conditions, which may vary spatially. However, without intermediate paleoprofiles that define how the knickpoint passed, exponent values from the optimization are ambiguous because they may include effects from both the dynamics of knickpoint propagation and the processes incising exposed bedrock. This uncertainty highlights the need to understand how bedrock knickpoints behave in rivers.

The exponents m and n are not uniquely determined by incision patterns of the Californian and Japanese profiles. This ambiguity prevents a definitive comparison of K values between these regions and the Australian and Kauai end-member incision laws. Consequently, we calculate K using both end-member laws and discuss the implications but caution that these conclusions depend strongly on which incision law is applicable.

If our examples of Californian and Japanese channels incise according to the Australian end-member, K decreases as general rock strength increases. Granitoids and metasediments in Australia have low values (10^{-6} - 10^{-7} $m^{0.2}$/yr), volcaniclastic rocks in California have intermediate values (10^{-4} - 10^{-5} $m^{0.2}$/yr), and mudstones in Japan have the highest K values (10^{-2} - 10^{-4} $m^{0.2}$/yr). Thus K varies from 10^{-7} to 10^{-2} $m^{0.2}$/yr in a manner consistent with a primary dependence on lithology (Table 2). This extreme range in K implies that the relaxation time of landscapes following tectonism should vary several orders of magnitude, depending on lithology. In an orogen with constant rock uplift rates, the extreme range of K also implies that exhumation rates should decrease substantially, at least for a time, as rivers reach crystalline rocks after incising less resistant overburden (volcaniclastics and sedimentary rocks). This large variation in K cannot be a result of age or paleoprofile elevation uncertainties (see Table 2), which are closer to 50-100% of the K value.

We also attempted to use median and average annual rainfall as a surrogate for a potential climate dependence of K, but found no relation. For instance, the extreme K values from the Australian rivers come from the same region.

Where stream power provides a robust measure of the processes incising bedrock and K is related to both the history of large discharges and the resistance of the rock to incision, we ought to be able to extrapolate locally calibrated K values to other basins with the same lithology, given that climate and tectonism are either invariant, or characterizable. For instance, incision of the Tumbarumba paleoprofile can be predicted well using the K from the adjacent Tumut River. We also have retrodicted river profiles for the Tumbarumba River using other Australian K values by reversing the sign of the stream power law, using the modern profile as the initial condition and adding elevation incrementally. The difference between K values for the Tumut and Tumbarumba Rivers is small enough that we can use them interchangeably to retrodict paleoprofiles. The rate constants from the Lachlan and Wheeo profiles are different enough that they are poor predictors of profile elevation by several hundred meters elevation over an elapsed period of 20 Ma. Thus if regional K values have a small variance, they can be used to predict or retrodict the long-term rate of bedrock incision on rivers without other evidence for incision rate. Retrodiction has the advantage of a well-known initial profile (modern), so if enough K values for different rock types and perhaps climate are known, the major uncertainty in retrodiction will become the base level history and perhaps the critical local slope controlling transitions from bedrock to alluvial reaches [e.g., *Montgomery et al.*, 1996].

7. Conclusion

Numerical integration of incision laws using ancient river profiles indicates two end-member models with strong and weak area dependence on long-term incision rate. Rivers with a strong area dependence on incision rate have stable base levels (e.g., Australian examples) and the stream power law has the form

$$\frac{dz}{dt} = KA^{0.3-0.5}S^{1.0}. \tag{3}$$

This expression is consistent with hydraulic geometry relations between width, area, and bankfull discharge, which predict that the exponent on area should be between 0.3 and 0.4. Rivers with a small area dependence on long-term incision rate are a second end-member (e.g., Kauai examples) and have base levels that lower rapidly enough to create knickpoints which propagate upriver, eroding in a fashion first described by *Seidl and Dietrich* [1992]. Incision in these systems is likely a threshold process in the sense that it is localized to areas of steepened slopes around the knickpoint where the alluvial cover is absent. Although incision at the knickpoint may follow the stream power law, the long-term spatial pattern of incision along the profile is a function of the rate at which the amplitude of the knickpoint decays upstream, while K_f represents the frequency and magnitude of knickpoint erosion. This combination can be expressed as

$$\frac{dz}{dt} = K_f A^{0.1-0.2}S^{0.0-2.0}, \tag{4}$$

where boundary conditions are implicitly included in K_f. Two critical and unresolved questions are (1) what sets the frequency, amplitude and decay rate of knickpoints that often characterize river incision in tectonically active regions and (2) how do such rivers evolve concave profiles?

Ancient river profiles from California and Japan have incision patterns that can be interpreted to support either model. If they follow the Australian end-member, these profiles can be used to calibrate a parameter (K) that characterizes the resistance of the bedrock to incision and the discharge history. This parameter would then vary over 5 orders of magnitude between mudstones and volcaniclastic rocks of Japan and California (10^{-2} - 10^{-5} $m^{0.2}$/yr) and granitoids and metasediments in Australia (10^{-6} - 10^{-7} $m^{0.2}$/yr), but its variation with climate remains unresolved. Such an extreme variation in K would require the transient rate of denudation and time constants for landscape evolution to vary greatly with lithology.

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